White Dwarf Flame Front Data Analysis

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Introduction:

We know very little about the physics of the beginning of the universe, however, we can learn a lot about it by looking "back in time." We can see "back in time" due to the speed of light never changing regardless of the reference point. When we look at very distant objects, we can see what they looked like a long time ago due to it taking so long for the light to get to us. This method allows us to study how objects evolved near the beginning of the Universe. One important phenomenon we can study is Supernovae, which is the death of a star. An average 1 solar mass star has roughly 3 sections to its lifetime: pre-main, main, and post-main sequences. Pre-main sequence is before it starts to burn hydrogen to helium. Main sequence is when it starts to burn hydrogen to helium. Post-main sequence is after it exhausts its burnable hydrogen supply and starts burning other elements. Stars burn primarily at the core, so the outer layers are mostly just gas that is conducting heat. During the post-main sequence, the star blows off its gaseous outer shell leaving a carbon and oxygen core behind; This core is called a white dwarf star. White Dwarf stars explode in a type of supernova called SN1a. This means that they explode leaving no star behind. This explosion is due to the White Dwarf accumulating too much mass (usually from another star it is orbiting) and exceeding the maximum mass it can withhold and still be stable; This mass limit is called the Chandrasekhar mass limit and is 1.44 solar masses. A white dwarf does not get bigger as it gains mass it only gets denser. These explosions are very difficult and expensive to model completely. So, it is easier to model parts of it at a time. We modeled a physical element of the supernova called the laminar flame. We did this to improve previous models of the same thing using modern codes.

Methods

What we wanted to model was the ADR equation for the flame. ADR stands for advection, diffusion, reaction. This equation models the laminar flame propagation and is only solvable by a computer because it is a nonlinear differential equation.

$$\frac{dE}{dt} + P\frac{\partial(1/\rho)}{\partial t} = \frac{1}{\rho}\frac{\partial}{\partial x}\left(\sigma\frac{\partial T}{\partial x}\right) + \dot{S}$$

FIGURE 1: The first term on the left describes the total gas energy relative to the time. The Second term from the left refers to the thermodynamics that describes how the flame heats up and expands or compresses with the change of density. The right side refers how the energy of the flame is diffused in nuclear reactions.

We used a stellar evolution code called MESA to model these flames. Using MESA, we first modeled a 15-solar mass and 1-solar mass star's evolution as a test to see if the code was working. Our next goal was to run the flame model. Once that had been made to work correctly. We ran the same code for different fuel density and fuel composition combinations. I was assigned densities from 1E+08 to 1E+09. We then recorded the flame speed and flame width for each run and plotted those values against each other. We found a regression for all the graphs and compared those formals to those done by other researchers like Timmes and Woosley (1992).

Results:

Each graph provided an exponential fit to the data but with different exponents and coefficients. The equations we got for the flame speed with a 50/50, 100/0, and 0/100 split between carbon and oxygen respectively are: $y = 0.046e^{2.03x}, y = 9.0861e^{1.53x}, y = 1E-07e^{3.40x}$. The equations we got for the flame width with a 50/50, 100/0, and 0/100 split between carbon and oxygen respectively are: $y = 4E9e^{-3.27}, y = 4E7e^{-2.8x}, y = 2E16e^{-5.05x}$

Flame Speed vs. Density

$y = 9.0861e^{1.5252x}$

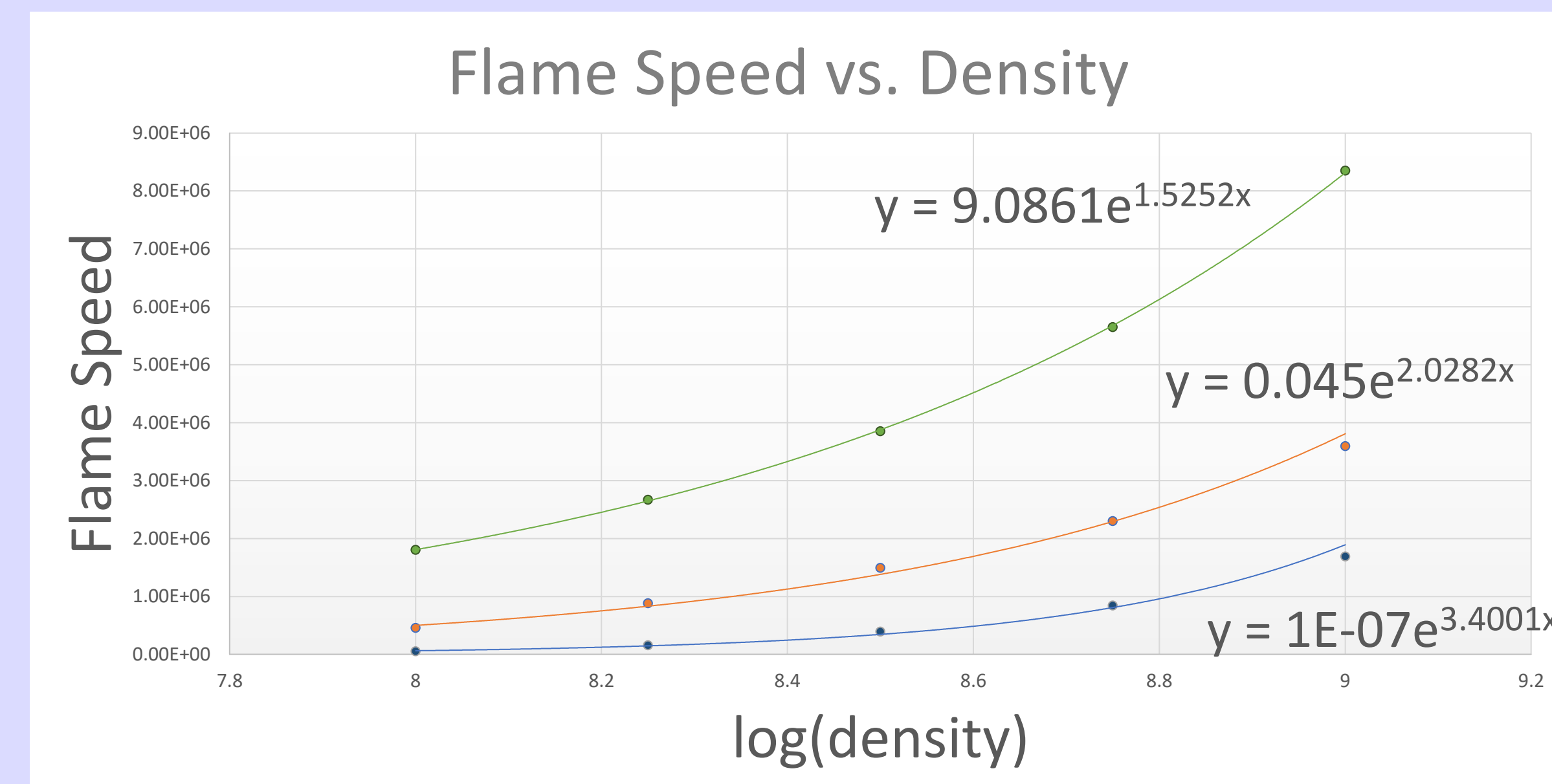$y = 0.045e^{2.0282x}$

$y = 1E-07e^{3.4001x}$

FIGURE 2a: These graphs each indicate the density of the flame compared with the flame speed. The graph has measurements for the speed the correspond to different densities that range from 1E+08 $\frac{g}{cm^3}$ to 1E+09 $\frac{g}{cm^3}$. The lines each correspond to a certain carbon and oxygen composition of the fuel (green is 50/50, orange is 100% carbon, and blue is 100% oxygen.) The equations above or below each line is the regression or fit of the data.
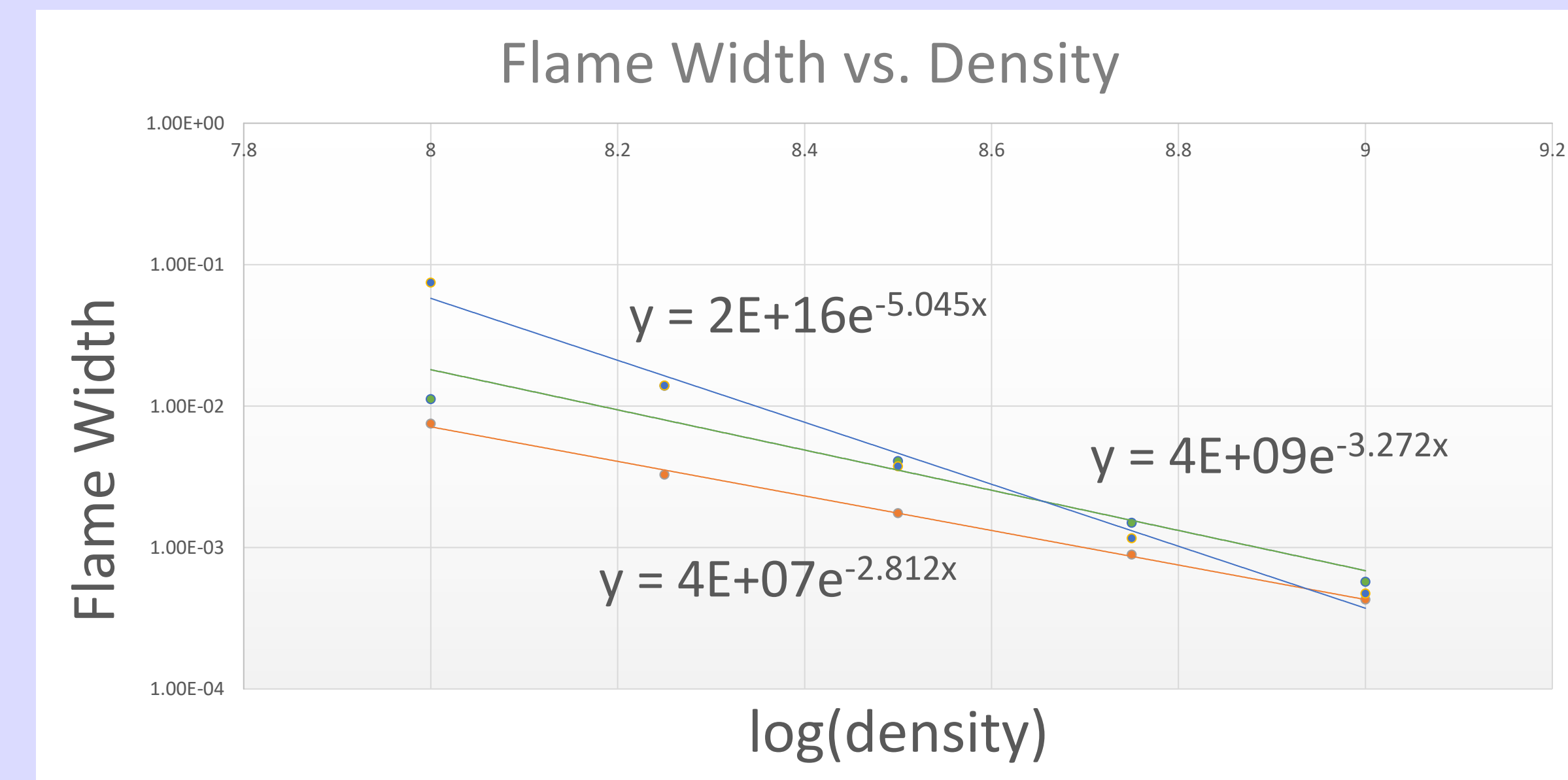

Flame Width vs. Density

$y = 2E+16e^{-5.045x}$

$y = 4E+09e^{-3.272x}$

$y = 4E+07e^{-2.812x}$

FIGURE 2b: These graphs each indicate the density of the flame compared with the flame width. The graph has measurements for the width the correspond to different densities that range from 1E+08 $\frac{g}{cm^3}$ to 1E+09 $\frac{g}{cm^3}$. The lines each correspond to a certain carbon and oxygen composition of the fuel (green is 50/50, orange is 100% carbon, and blue is 100% oxygen.) The equations above or below each line is the regression or fit of the data.

FIGURE 3a: (Below) This is a graph directly from that code that shows the log of the temperature (orange) and the log of the density (blue) compared to the flame's radius. The line being as sharply angled indicates that the flame is not interfering with other things.
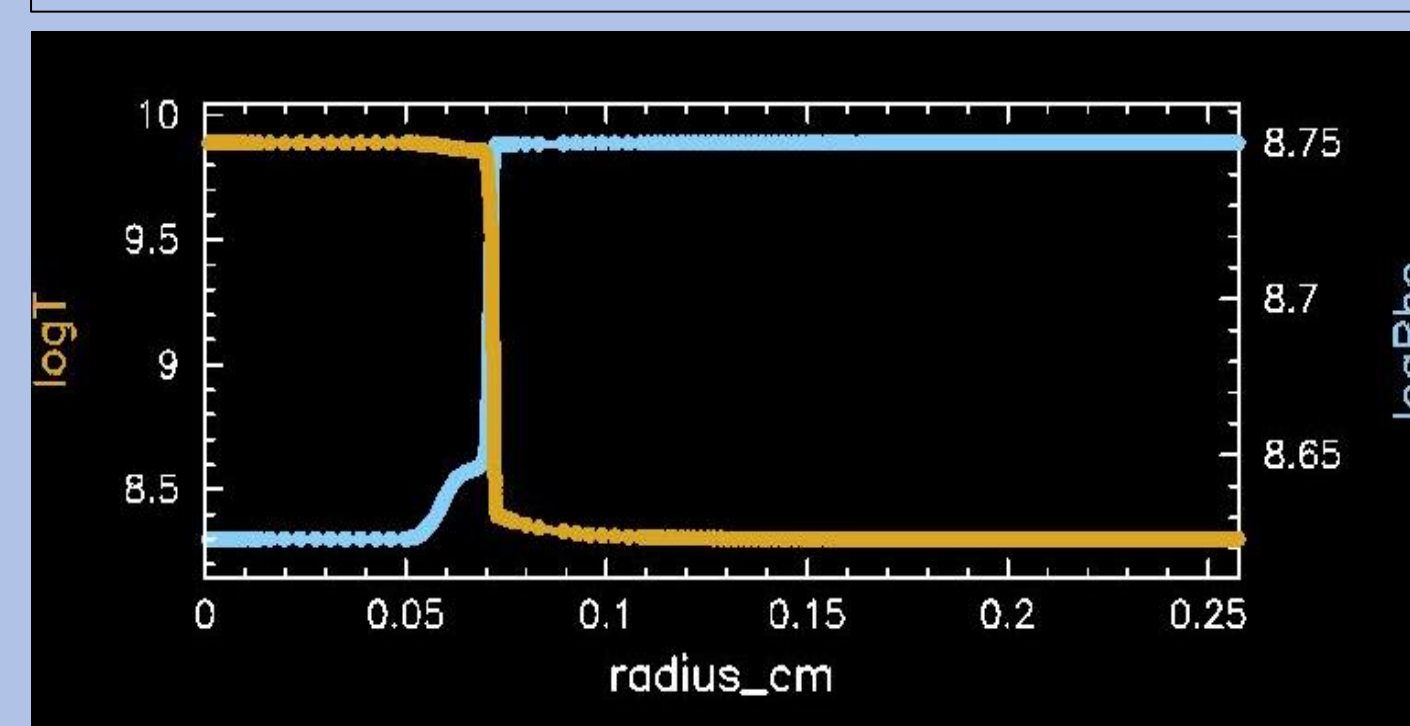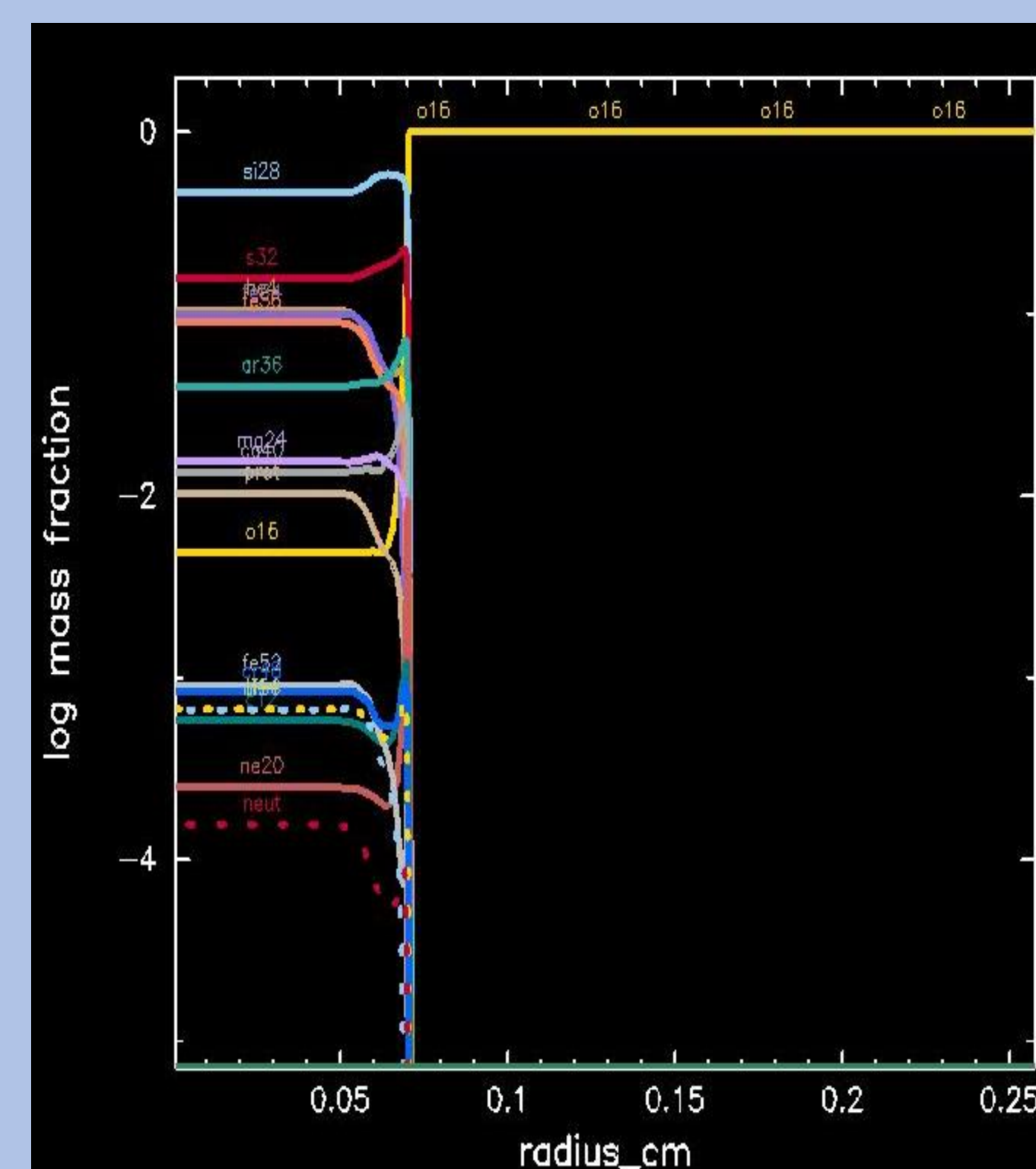

FIGURE 3b: (Right) This graph is directly from a MESA code run and shows the elemental composition of the flame over time. Each color indicates a different element.

Discussion:

The fits we collected were exponential which is a kind of power law. The fits collected by Timmes, and Woosley (1992) were also Power Laws. This shows that the data we collected, and they collected has the same type of regression. Since our data is accurate (we have more decimal places) it is likely an improvement from what was done in 1992. However just like they did we had quite a bit of error. This is shown in figure 5 and in the inconsistency in the points on the C50/O50 Flame width graph in figure 2b. There is still room for improvement due to the error being considerably large. Future research would be to try to refine this data by doing with more nuclear networks instead of just this simple one
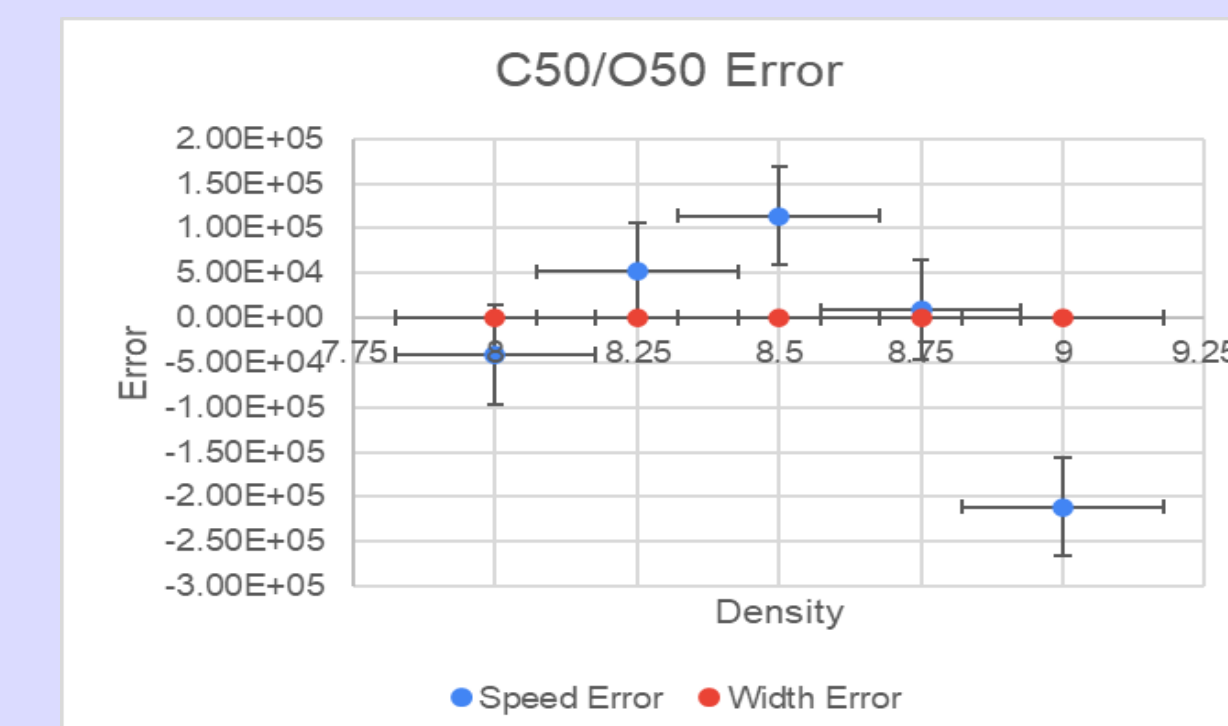

C50/O50 Error

FIGURE 5a: This graph shows the error of the 50/50 Carbon Oxygen graphs.

C100/O0 Error

FIGURE 5b: This graph shows the error of the 100% Carbon graphs.

FIGURE 5: These graphs show the amount of error for each point of each of the runs. Each graph corresponds to a different composition. The red dots represent the flame width MESA measurement minus the fit, and the blue dots represent the flame speed MESA measurement minus the fit.

FIGURE 4a: (Top Half) This is a graph from the running MESA code that shows the change in flame width with the stars age.

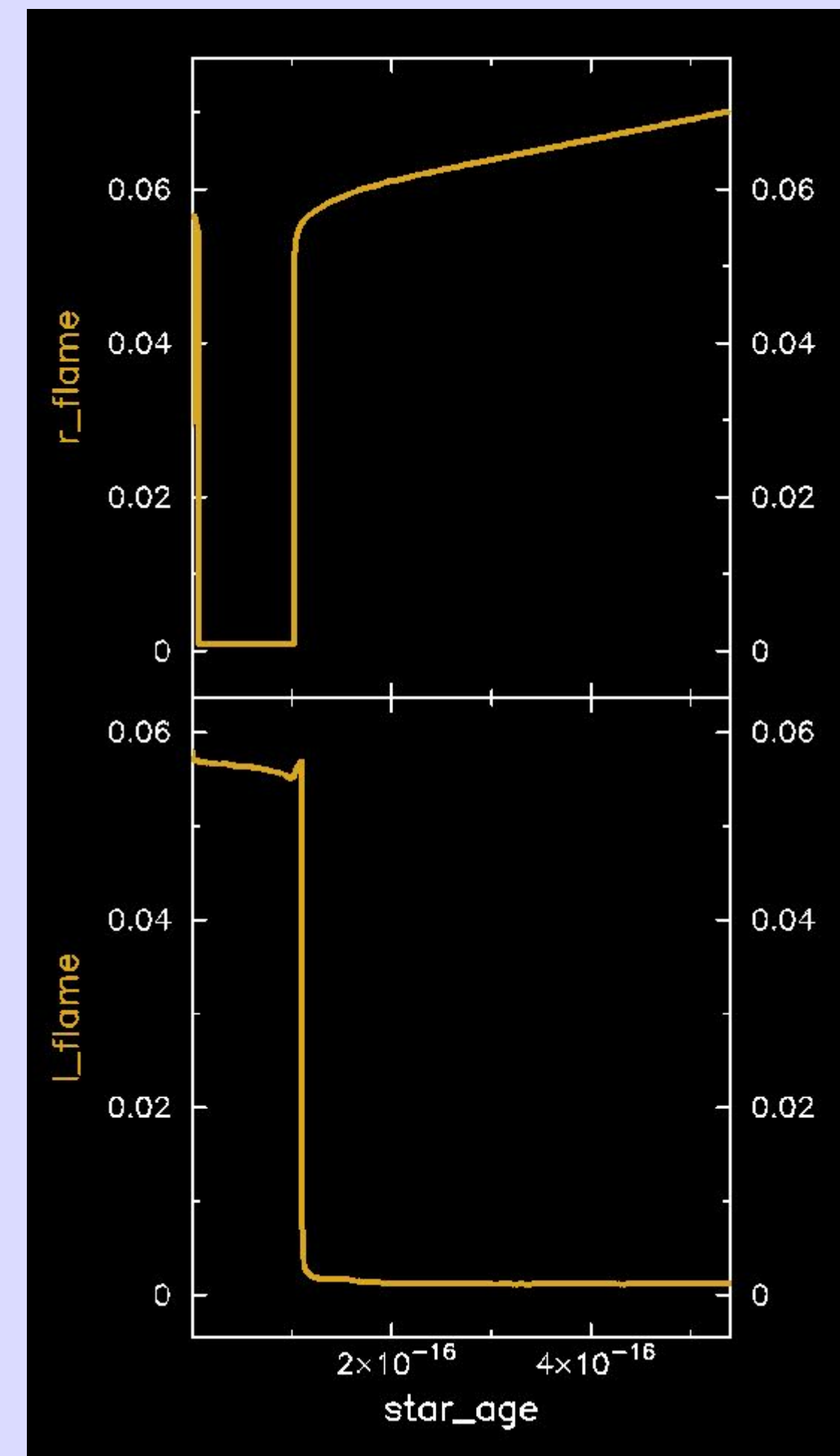**FIGURE 4b**: (Bottom Half) This is a graph from the running MESA code that shows the flame speed as function of the star's age.
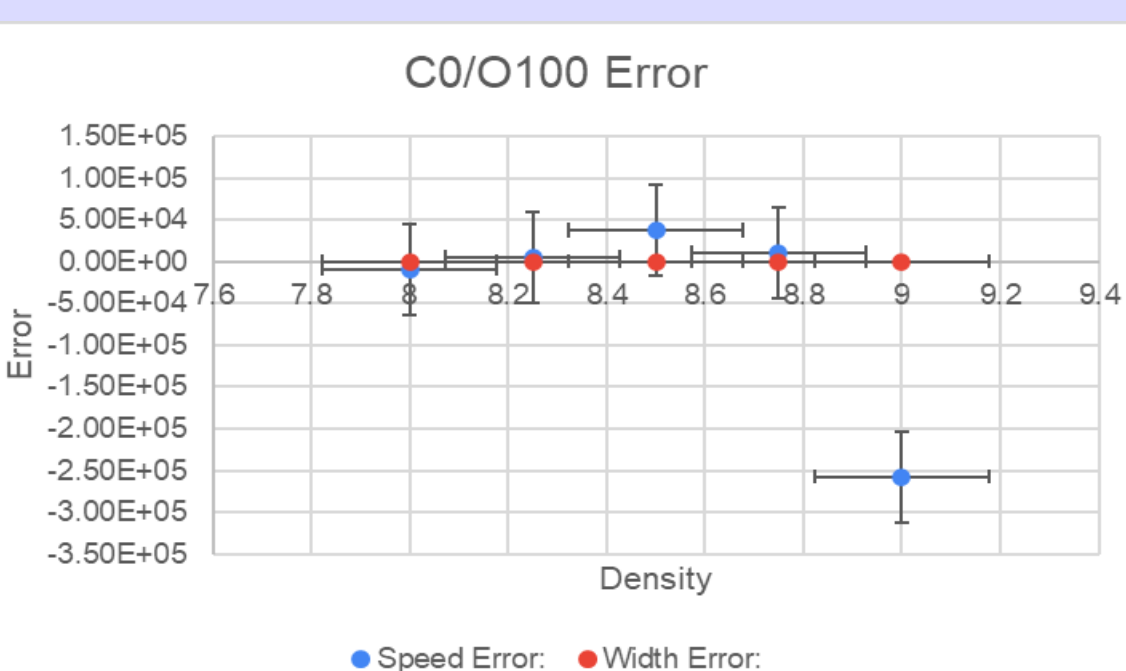

C0/O100 Error

FIGURE 5a: This graph shows the error of the 100% Oxygen graphs.

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