Numeric Properties in Type Ia SN Flames

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Aims

Type Ia supernovae form at the end of the life cycle of low mass stars. This occurs when they satisfy certain criteria such as their mass during late stages of their evolution. Attempts to model an SN Ia have been made for decades. However, these astronomical events are incredible in size, and have complex physical systems that are challenging to study even for the larges computers. To overcome this limiting roadblock, we are using MESA (Modules for Experiments in Stellar Astrophysics), which models a proportion of a stellar mass, rather than an entire star. With it one can predict theoretical properties of stars including the speed of burning fronts, burning rates, and resulting composition. In this project we use MESA's model flame setup and systematically vary the density and composition of stellar fuel to obtain an approximate formula for the laminar flame speed. We will also compare our results with that of previous studies.

Methodology

A thermonuclear flame is modeled for a given fuel density and composition. We approximate dependance of the flame speed, v_f, and flame width, δ_f, based on a range of densities characteristic of the white dwarf interior. While a flame's properties are a function of both density and composition, this work will look at a single composition of 50% carbon and 50% oxygen, making density our primary parameter space. Following procedure described by Timmes & Woosley (1992; hereafter TW92), and in the research of Josiah Schwab, R. Farmer, and F. X. Timmes (2020; hereafter SFT20). These two previous research publications will be used as the basis of comparing our results to that of existing data, for which we will attempt to offer suggestions. As we alter the density of these flames using MESA's conductive flame model, we produce new flame speeds, v_f, and flame width (δ_f) approximated by the formula F(D,C) where D and C are the fuel density and composition, respectively. We then compare our results to that of those from TW92 and SFT20.

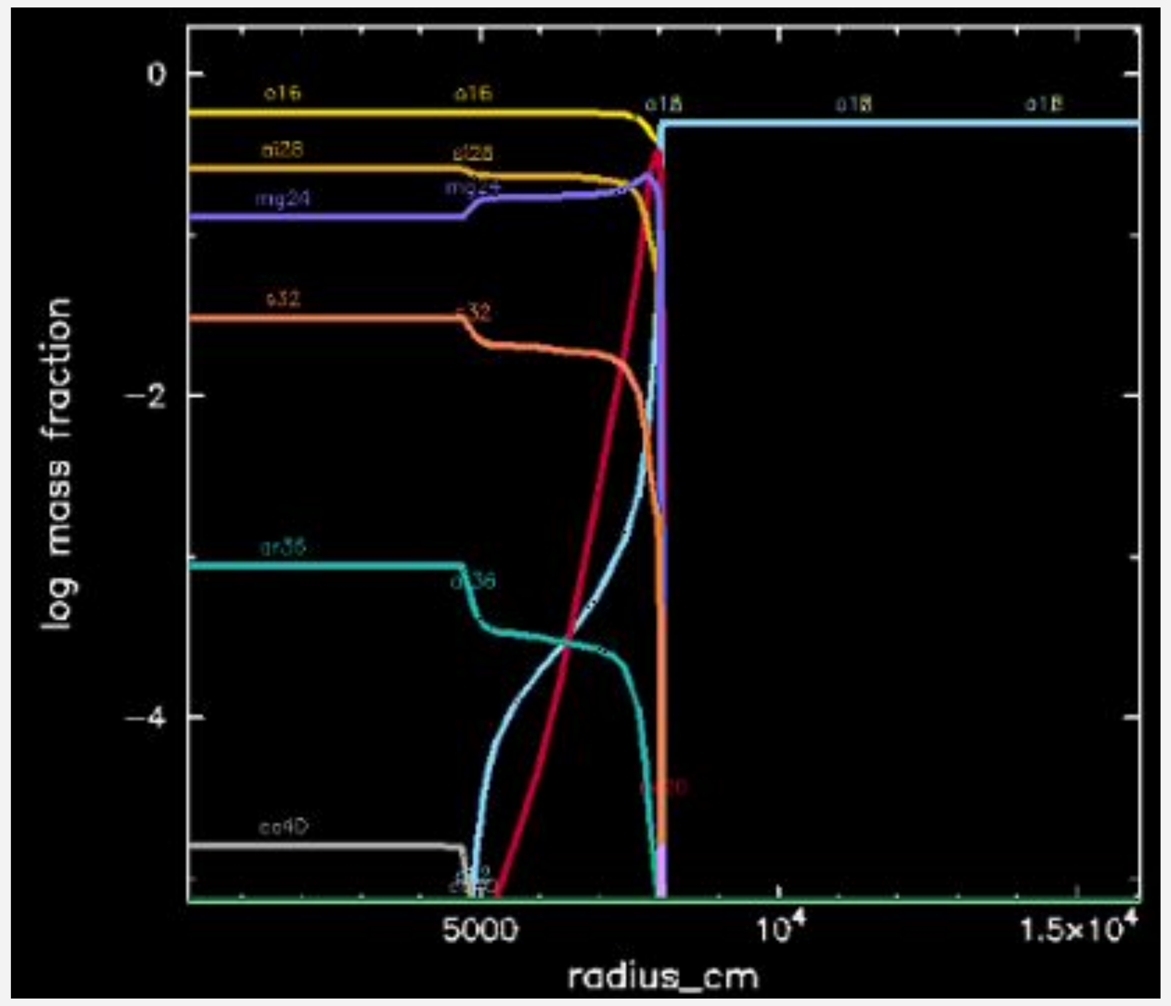

Fig. 1: MESA flame model of a 5×10^6 g/cc density flame. This is a 50/50 Composition model with a Mass of 5×10^{17} grams. In this we can see the x-axis radius, wherein the left side of the graph represents being closer to the center, and the outer right side represents elements closer to the surface. In this we can see silicon, magnesium, and other elements forming and shifting location based on their mass.

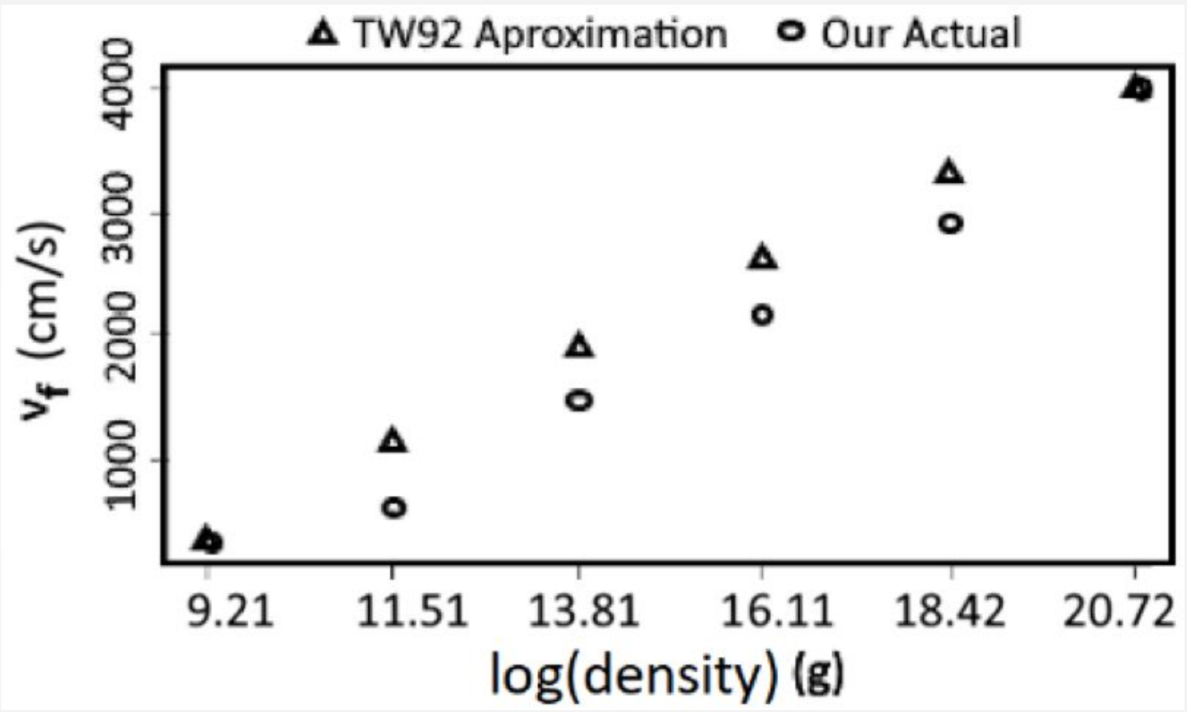

Fig. 2: v_f, graphed as a function of log(density). We can see a relationship with our data in those two parameters, and we can compare that alongside the TW92 data. Note it is not a perfect linear relationship, particularly around lower density models.

Results

Using MESA we modelled flame evolution for a range of fuel densities between 1×10^6 grams and 1×10^9 grams. To demonstrate a stable star we then modified the mass until the module successfully completed to the halfway point as shown in Figure 1. Once the model completes it provided flame speed v_f, and width, δ_f, which we then can observe the flame speed as a function of density as shown in Figure 2.

Figure 2 shows the relationship between flame speed and density in our model (circles) and that of TW92 (triangles). The 'TW92' data marked with triangles approximated in mathematical formula presented in Figure 3. This is only accurate to about 10% within our density range. Note that our results are not reflective of a purely linear relationship at these low densities, whereas TW92 appears more linear when compared in Figure 2. Similarly, we can review the graph produced from TW92, shown in Figure 4, where the densities they used begin at 1×10^9 and higher. At those higher densities TW92 reflects a nonlinear relationship, similar to to what our results indicate within Figure 2. To take our analysis of the data further we can also look at the SFT20 fit test, figure 5, which graphs the difference of fit between the data in that research alongside TW92.

$$v_{\rm cond} = 92.0\left(\frac{\rho}{2\times10^9}\right)^{0.805}\left[\frac{X(^{12}\mathrm{C})}{0.5}\right]^{0.889}\ \mathrm{km\ s^{-1}}$$

Fig. 3: TW92 Formula to approximate flame speed for a 50/50 carbon/oxygen composition.

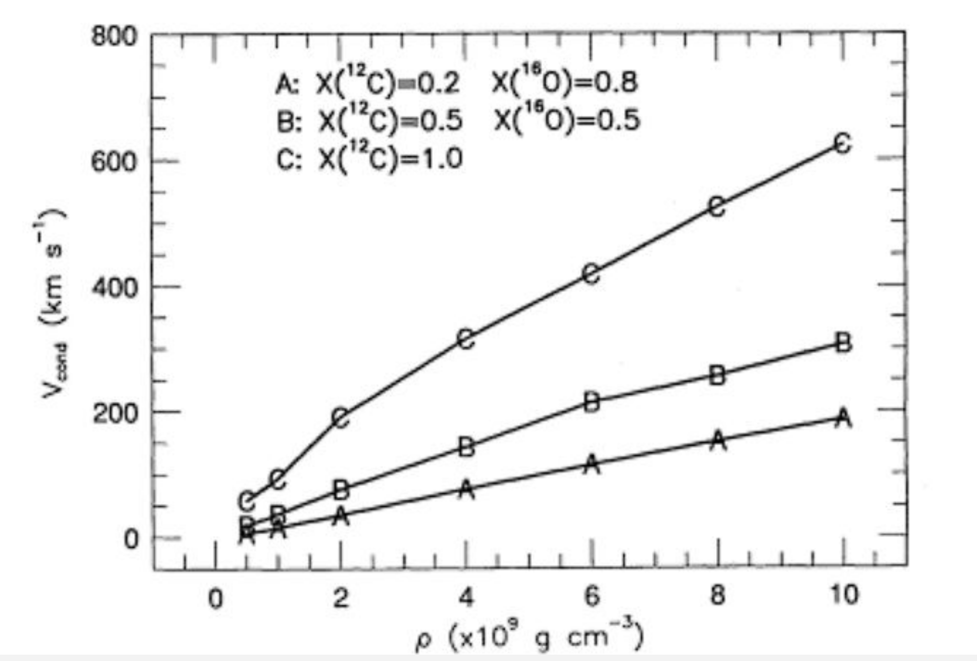

Fig. 4: Conductive flame speed as a function of density according to TW92 data. The different lines represent differing mixtures of compositions of stars, and show how the density and composition result in differing flame speeds.

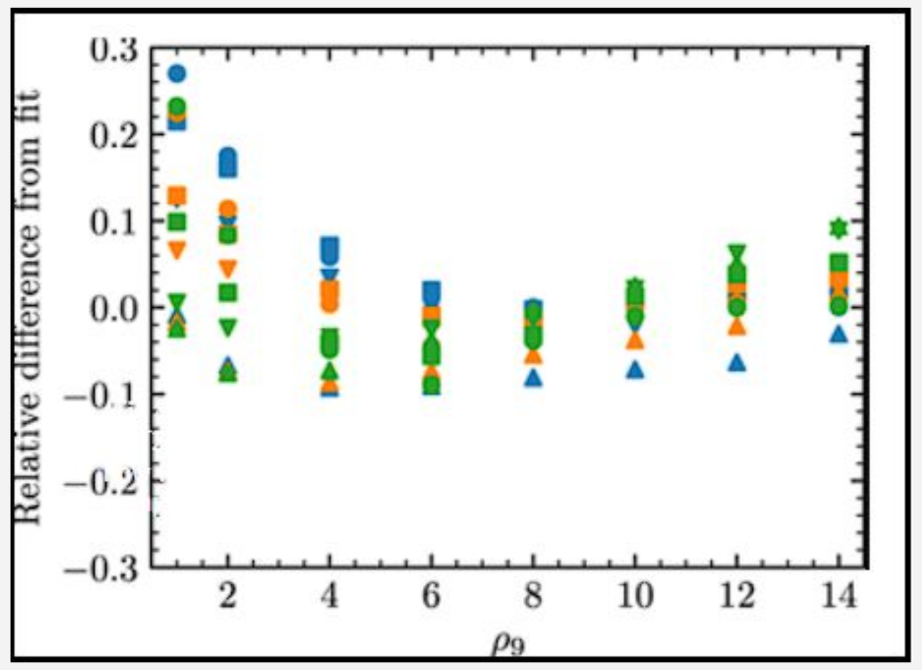

Fig. 5: SFT20 Difference of fit between SFT20 data and TW92 data by density - we can see how well these modules do or do not coincide with one another.

Discussion

TW92 does not contain much data of lower density stars, leaving us only the formula within the paper to produce the comparative model in Figure 2. The trend of their approximations appear more linear, but should instead reflect a nonlinear relationship like the one depicted both in our data indicated in circles, and also like the data that TW92 presents in Figure 4. We expect a strong nonlinear flame speed dependence on the fuel density at low densities, suggesting the presented approximations have some amount of merit worth investigating with further research. With MESA capable of calculating lower density stars relatively quickly and without arduous computational power, and with differing results stable stars properly expanding out to the halfway point such as the one displayed in Figure 1, we can continue looking into other alterations of these models to continue learn more about the conductive flame. It's important, however, to note that this research has many restrictions. These approximations are very introductory, and best used for exploratory interest rather than any sort of formal proposition. However MESA's conductive flame can successfully approximate properties for laminar flames at lower densities, though these will not directly follow TW92's formula for finding these flame speeds.

Conclusions

Further analysis and testing beyond a handful of models would be necessary. Ideally additional iterations would be used to create more complete data (perhaps, something to automate looping through various densities and storing the output for its flame velocity). This future study would also benefit from testing the various compositions of stars, similar to how SFT20 and TW92 utilized different compositions in their own data. Efforts to complete different iterations in this fashion would allow us to see how changes in the oxygen/carbon levels affect flame speed, and should be relatively simple once automated via a script thanks to MESA's already somewhat low computational effort.

References/Acknowledgments

- Thank you to James Hugglestone for his insights
- Thank you to the UROP team for supporting and providing this opportunity.

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