Modeling the Optimal Reaction Norm for Inducible Prey Defenses

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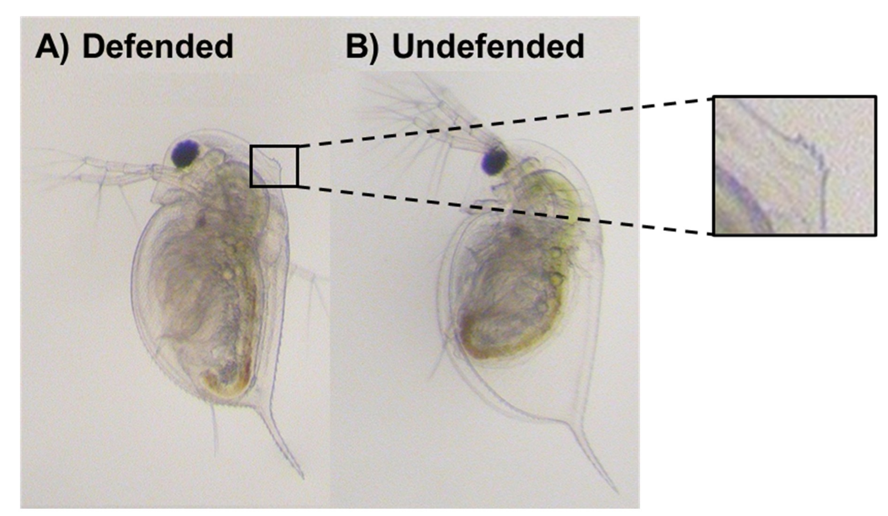
What are Inducible Defenses?

Induced Defenses are defenses that prey express in response to the threat of predation. These defenses can behavioral, chemical, or morphological

An octopus squirts black ink into the water (Photo Credit : Vittorio Bruno/Shutterstock)

Lima beans, when they are being eaten by spider mites, increase their production of VOCs

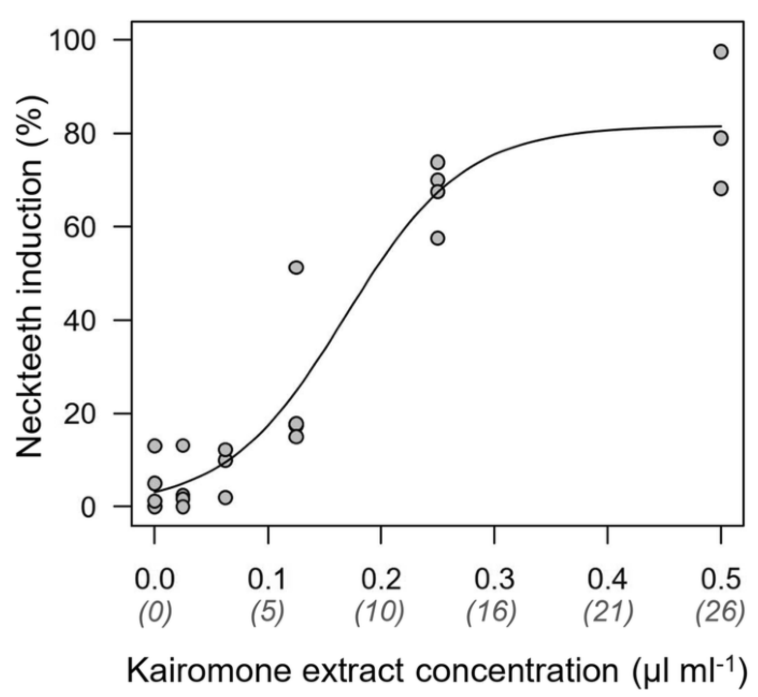


Daphnia grow spikes (neckteeth) when predators are around (Photo Credit: Sam Paplauskas et al.)

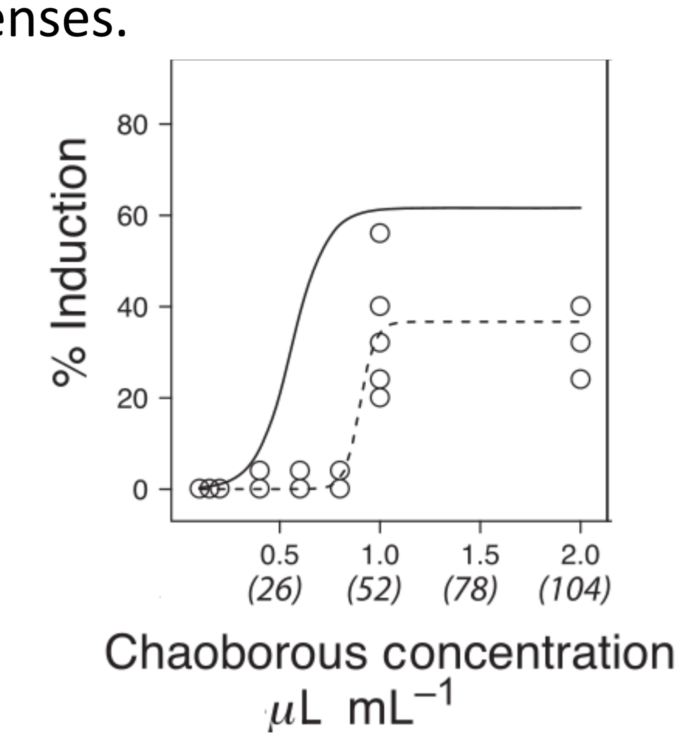
These defenses can help protect prey from predators, but they have costs, including increased energy expenditure and reduced opportunity to find food or a mate.

What are Reaction Norms for inducible Defenses?

The reaction norm is the relationship between an individual's prey defense level and the number of predators it senses.



The reaction norm of the induced defense of Daphnia where kairomone extract concentration is used as a measure of predator density (Sperfeld et. Al. 2020)



The reaction norm of the induced defense of Daphnia where kairomone extract concentration is used as a measure of predator density (Hammill et. al. 2020)

Question: What is the optimal reaction norm that prey species should have in response to predators?

Conclusion: The reaction norms have two possible shapes depending on the trade-off between reproduction and defense. Continuous reaction norms occur when the costs of defense accelerate faster than the benefits whereas threshold reaction norms occur when the benefits accelerate faster than the costs

Future work: My Future work focuses generalizing these results to models where r(α) and a(α) are other non-linear functions of the trait and models without a Lotka-Volterra form.

$$\frac{dx}{dt} = x[r(\alpha) - a(\alpha)y]$$

- Represents the change in prey with respect to time ($\frac{dx}{dt}$)
- Prey Density (x)
- Function representing reproductive success of prey (r)
- Alpha is the defense rate of the prey (α)
- Function representing the success of predator attacks (a)
- Predator Density (y)

I solved the Euler-Lagrange equation to find first and second variation

Through approaches in Calculus of variation

The first variation can be set to 0 to find an equation for y (predator density) dependent on α (the defense rate of the prey)

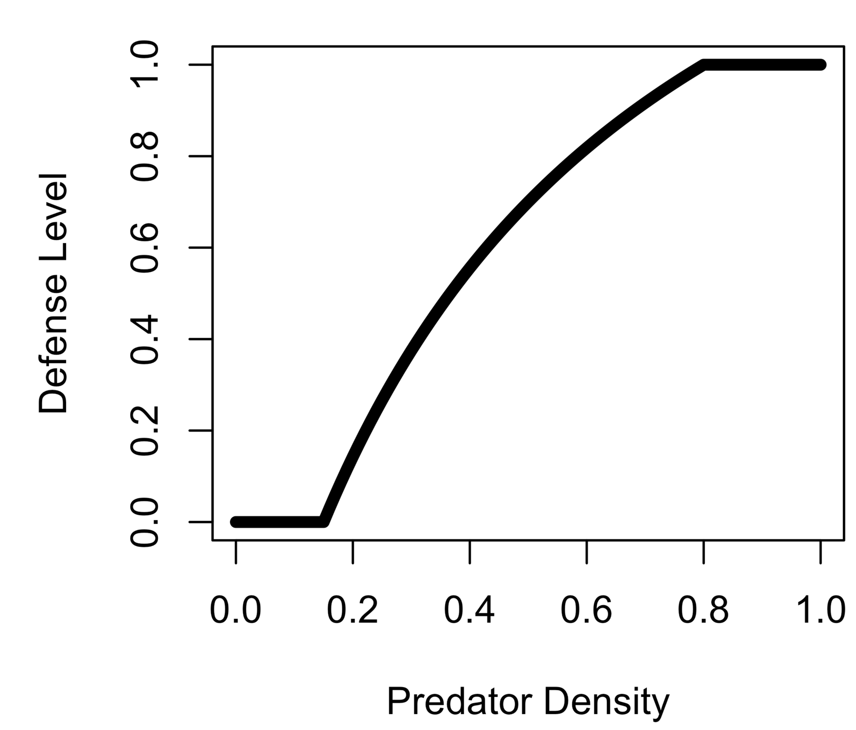
The second variation determines whether it is a function that maximizes or minimizes prey fitness

Because the rate of induction is dependent on density of predators having an equation in the form of α=... is needed to find the shape of the reaction norm

$$y = \frac{r'(\alpha)}{a'(\alpha)} \xrightarrow{\text{Implicit Differentiation}} \frac{d\alpha}{dy} = \frac{a'(\alpha)}{r''(\alpha) - a''(\alpha)y}$$

If we take $r(\alpha) = r_0 + r_1\alpha + r_2\alpha^2$, $a(\alpha) = a_0 + a_1\alpha + a_2\alpha^2$ Then $\alpha = \frac{ya_1 - r_1}{2r_2 + 2a_2 y}$

There are two general shapes of the optimal reaction norm:



The continuous Reaction norm

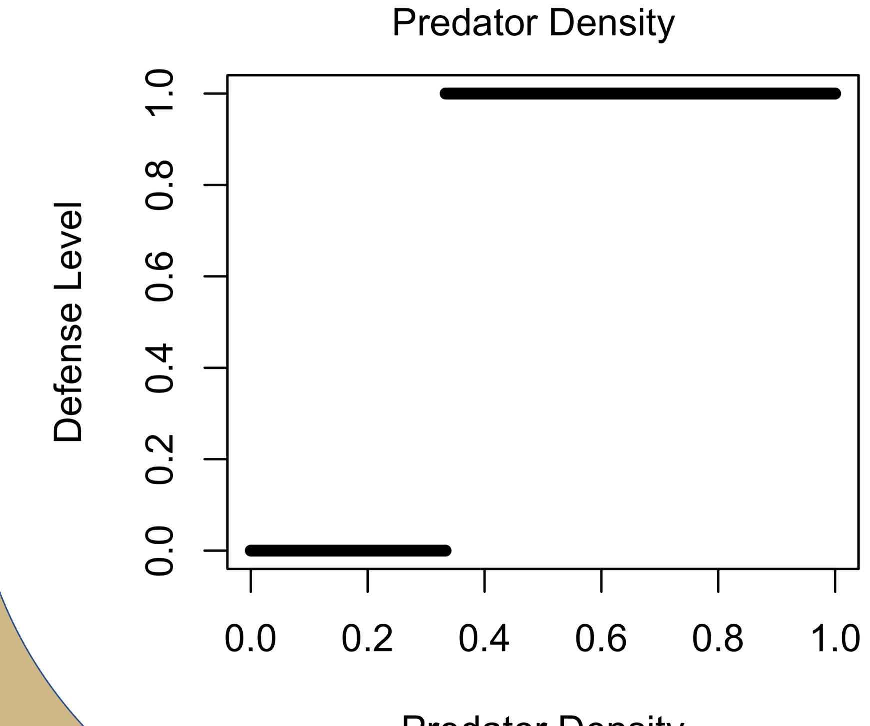
This occurs when the cost of defense accelerates faster than the benefits of defense.

For example:

$r_2 < 0$ which represents accelerating costs and

$a_2 > 0$ which represents decelerating benefits

Produces a continuous reaction norm



The Threshold Reaction norm

This type of reaction norm occurs when the benefits of defense accelerate faster than the costs of defense

For example:

$r_2 > 0$ which represents decelerating costs and

$a_2 < 0$ which represents accelerating benefits

Produces a threshold reaction norm