

Noise In Quantum Simulations

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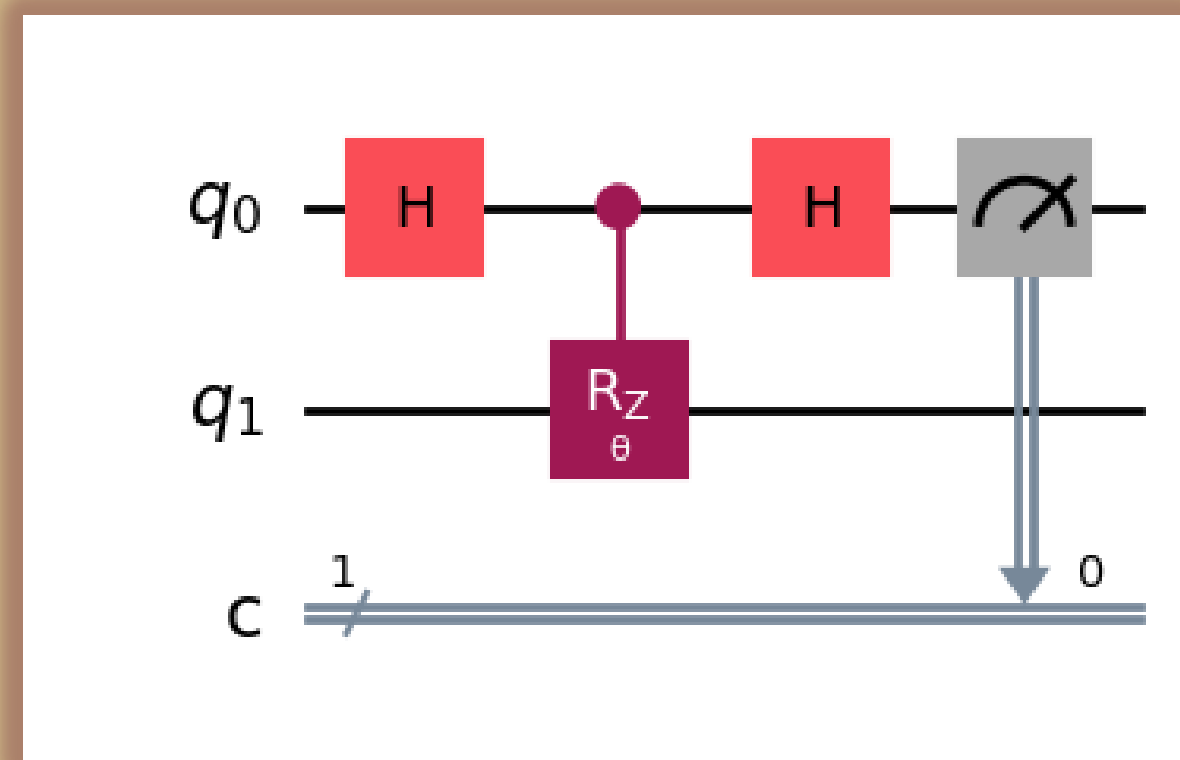
Abstract

Quantum computers are exciting because they can solve some tasks faster than traditional computers. They achieve this by utilizing quantum effects such as superposition and interference, which allow them to execute many outcomes at once instead of individually. However, current quantum computers are imperfect and are impacted by noise, which limits how accurately they can compute. In this study, we measure and analyze noise within trapped-ion quantum computer simulations. We ran a simple quantum circuit known as the Hadamard test on simulated IonQ quantum computers both with and without hardware noise. Our circuit was run multiple times at different shot levels. We compared measured results to exact theoretical values and used statistical methods to determine whether differences were due to regular measurement fluctuations or because of hardware noise. It was found that at lower shot counts below around 16,000 shots, deviations were in margin with expected statistical noise. However, at shot counts above around 64,000 shots, the differences grow larger than what is accounted for in statistical fluctuations. This shows that hardware noise becomes visible once statistical error is reduced. These results provide a simple and effective method for statistically separating hardware noise and statistical noise in simple quantum circuits. This method can be expanded upon in future studies involving real quantum hardware along with more complex circuits.

Background

The Hadamard Test

- Quantum computers utilize the expectation value of operations which is the average outcome obtained when a quantum operation is applied multiple times to the same quantum state. A standard method of measuring expectation values is with a Hadamard test. This circuit prepares a qubit in superposition, applies a quantum operation, and then uses interference to convert information into measured probabilities.
- This test extracts the real part of the overlap between a quantum state $|\psi\rangle$ and the same state after applying a unitary operator $U: \langle\psi|U|\psi\rangle$. This expression represents how similar the quantum state is to itself after applying the unitary operator.
- For single-qubit rotations $U = R_z(\theta)$ which is a rotation around the z-axis where θ is an angle parameter that is varied between 0 and 2π for which the theory predicts $\cos \frac{\theta}{2}$ expectation value as the real part of our overlap from before. The circuit is shown below where in sequence our top qubit q_0 is our ancilla qubit our bottom qubit q_1 is our system qubit and we have H on ancilla, R_z, H on ancilla, and our measure ancilla:



Considering Noise

- Because quantum measurements are probabilistic, even though probability may be fixed, measurements that are repeated will fluctuate between 0 and 1 for qubits.
- To find expectation values measurements are repeated many times, and each repetition is called a shot.
- Finite number of shots produces statistical fluctuations, specifically uncertainty that decreases $\propto \frac{1}{\sqrt{N}}$ where N is the number of shots.
- Quantum hardware contains other factors that contribute to deviations not accounting shot noise.
- Analyses of interference patterns are often qualitative than quantitative
- It is not completely clear if increasing shots improves agreement with theory without tests.

Agreement with Theory

- To see if deviations are aligned with expected statistical fluctuations we take a z-score: $z = \frac{\text{Residual}}{\sigma}$ where residual is the difference between measured value from theory and σ is the expected statistical fluctuation. If $|z| \approx 1$ then the deviation is consistent with statistical noise. If $|z| \gg 1$ then the deviation larger than what we would find from statistical noise solely.
- If we want to find this similarly but for all angles we use a reduced chi-square test:

$$\chi^2_{\text{red}} = \frac{1}{N_{\text{eff}}} \sum \left(\frac{\text{Residual}}{\sigma} \right)^2$$

Where N_{eff} is the number of data points. If $\chi^2_{\text{red}} \approx 1$ deviations match expected statistical noise. If $\chi^2_{\text{red}} > 1$ then deviations are larger than statistical noise. If it increases with the number of shots then hardware noise exists.

Methods

Subjects

- Quantum circuits executed using IonQ simulation
- Ideal (noise-free) and "aria-1" noise simulators

Materials/Measures

- Single-qubit Hadamard test using $U = R_z(\theta)$
- Nine sampling values of θ from 0 to 2π
- Shot counts: 4k, 16k, 64k, 160k, 640k
- Expectation values accounted for:
 - $R_{\text{measured}} = 2p - 1$ where p is the probability of measuring the ancilla in 0 state
- Residual: $R_{\text{measured}} - \cos \frac{\theta}{2}$

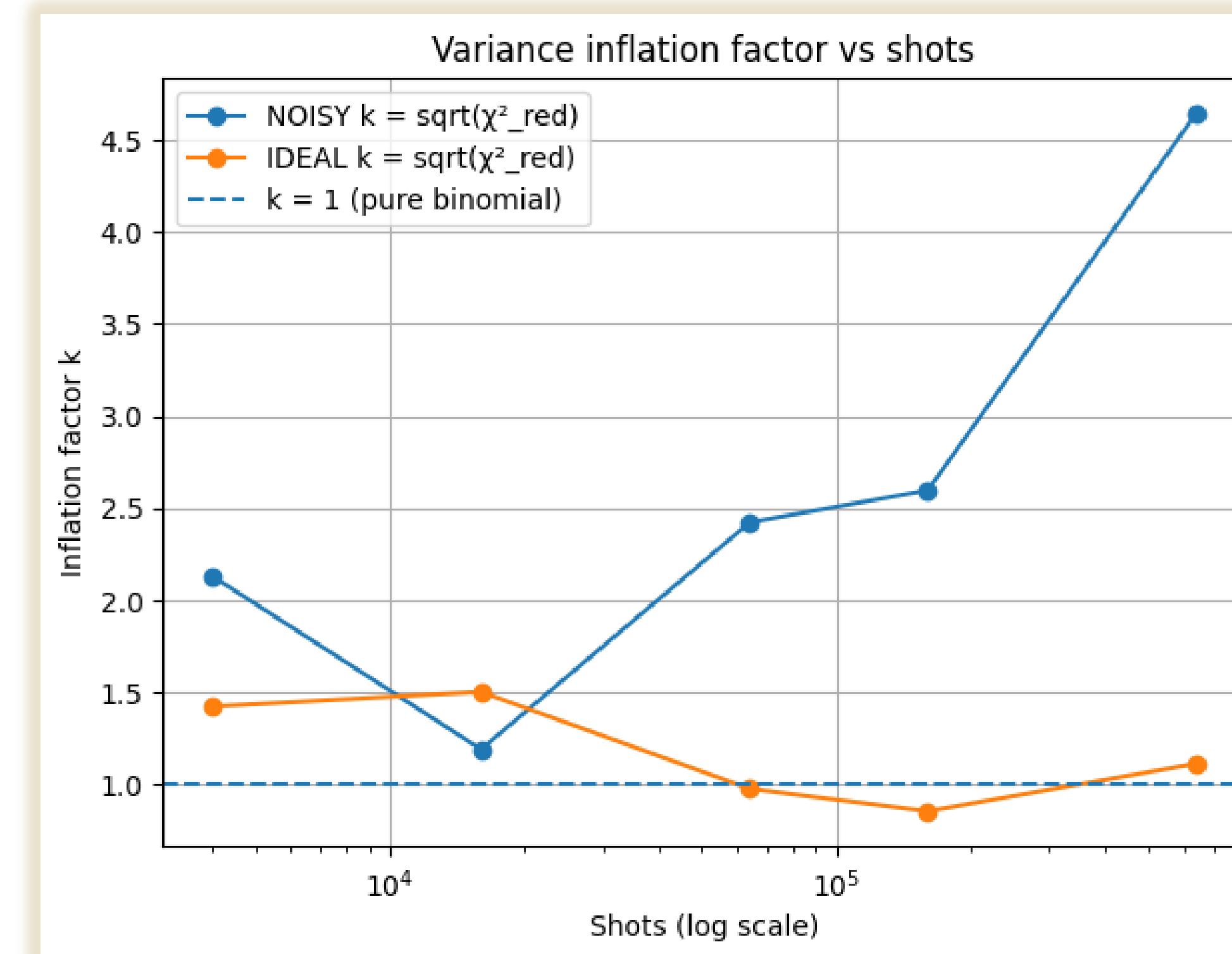
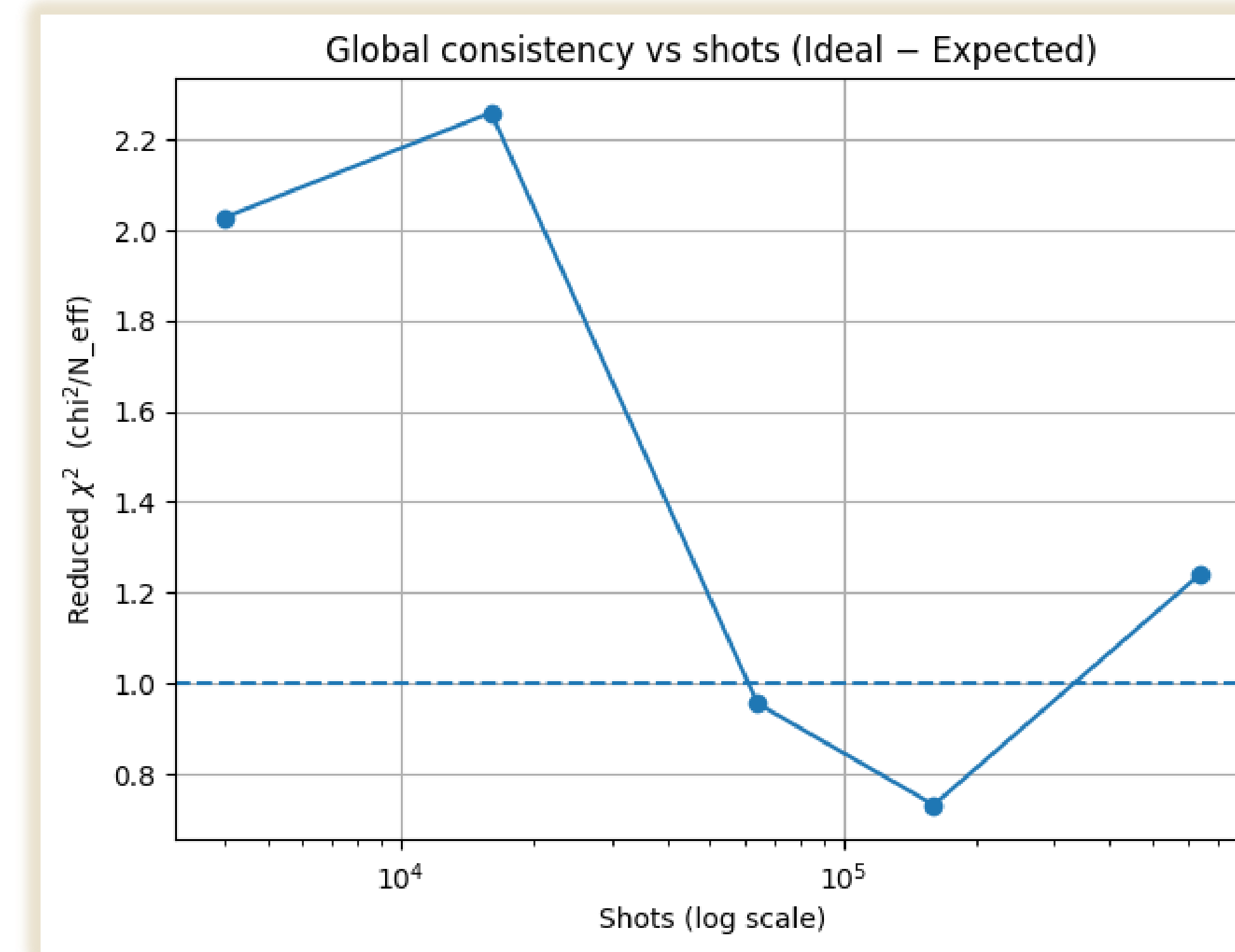
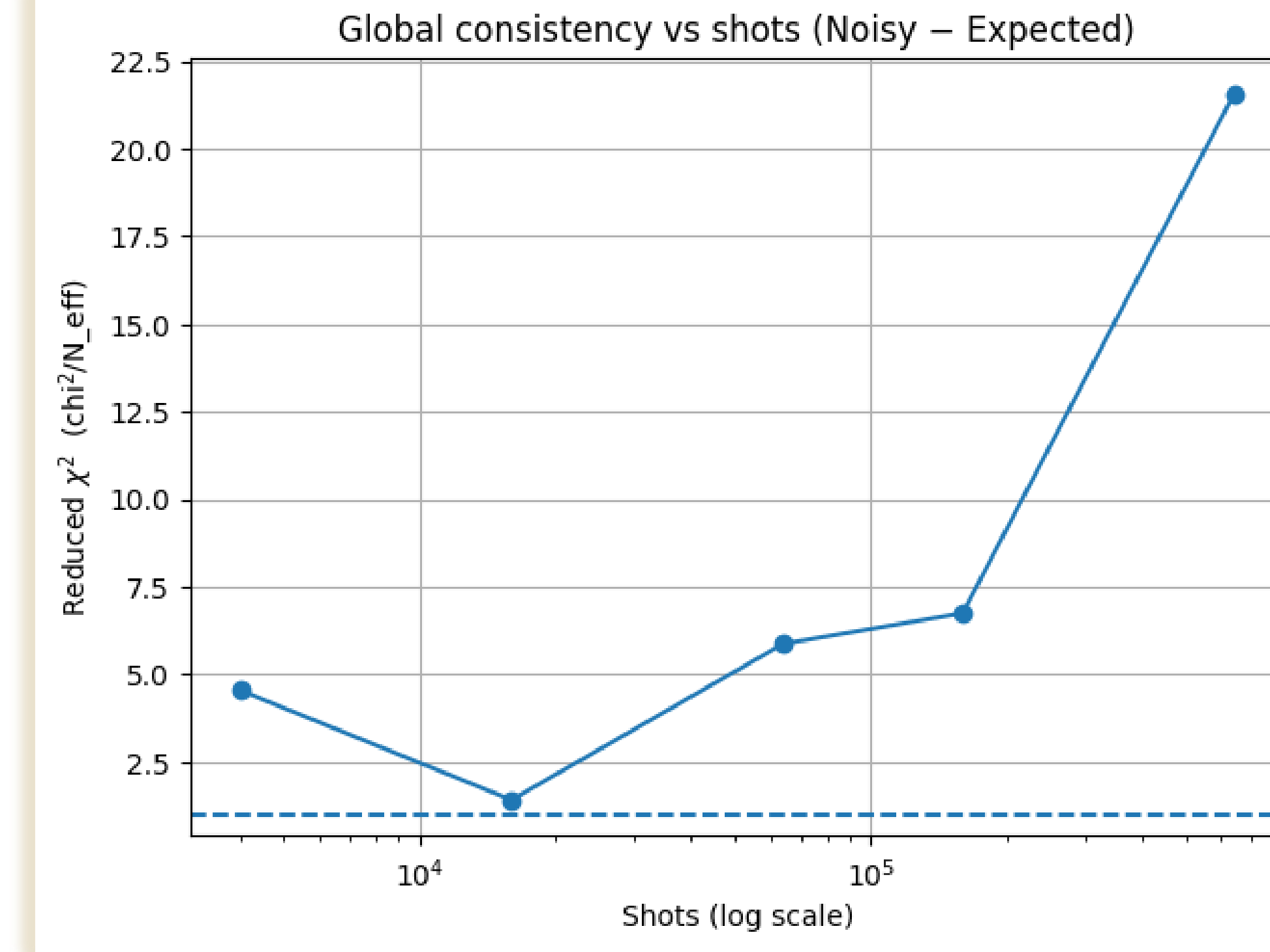
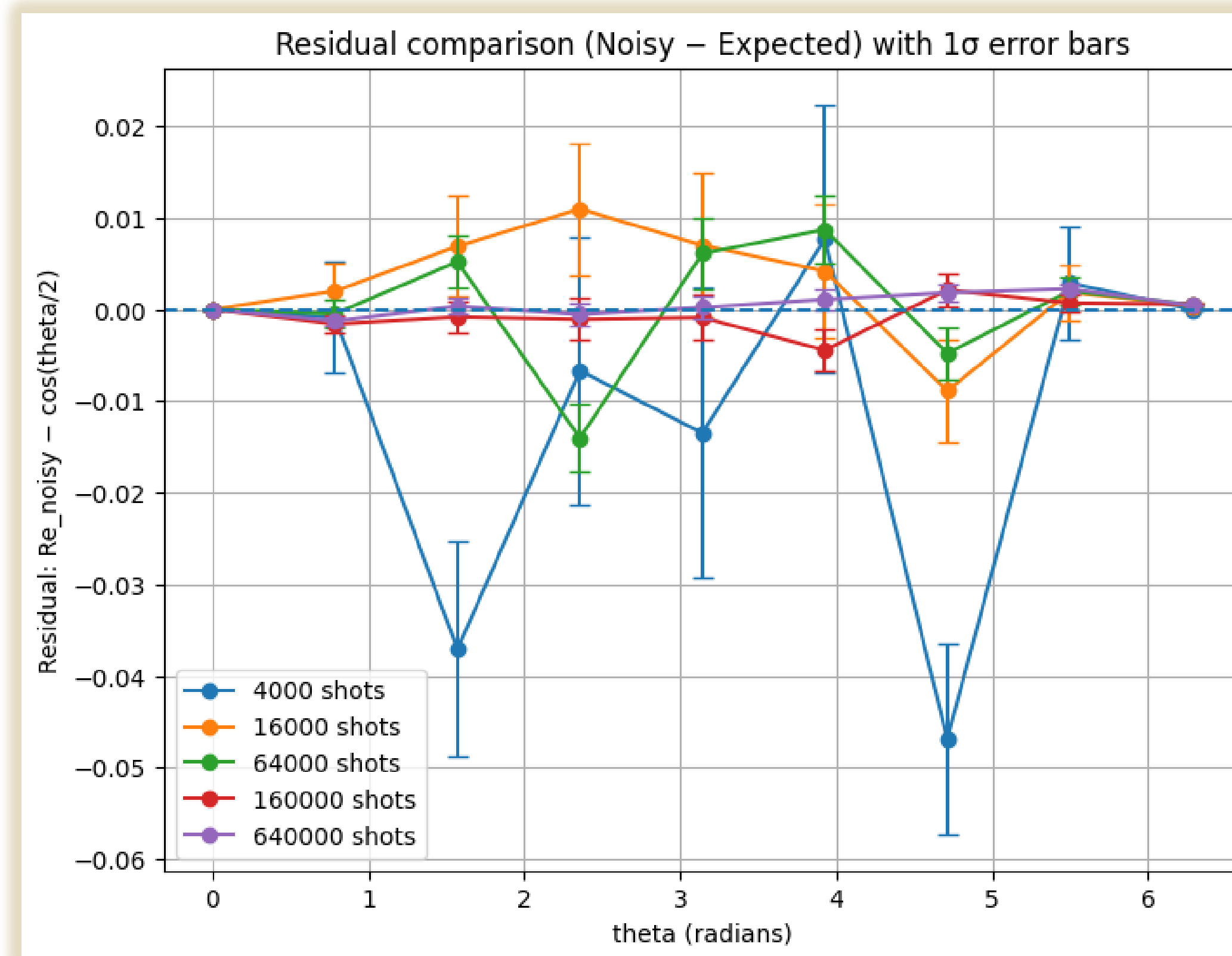
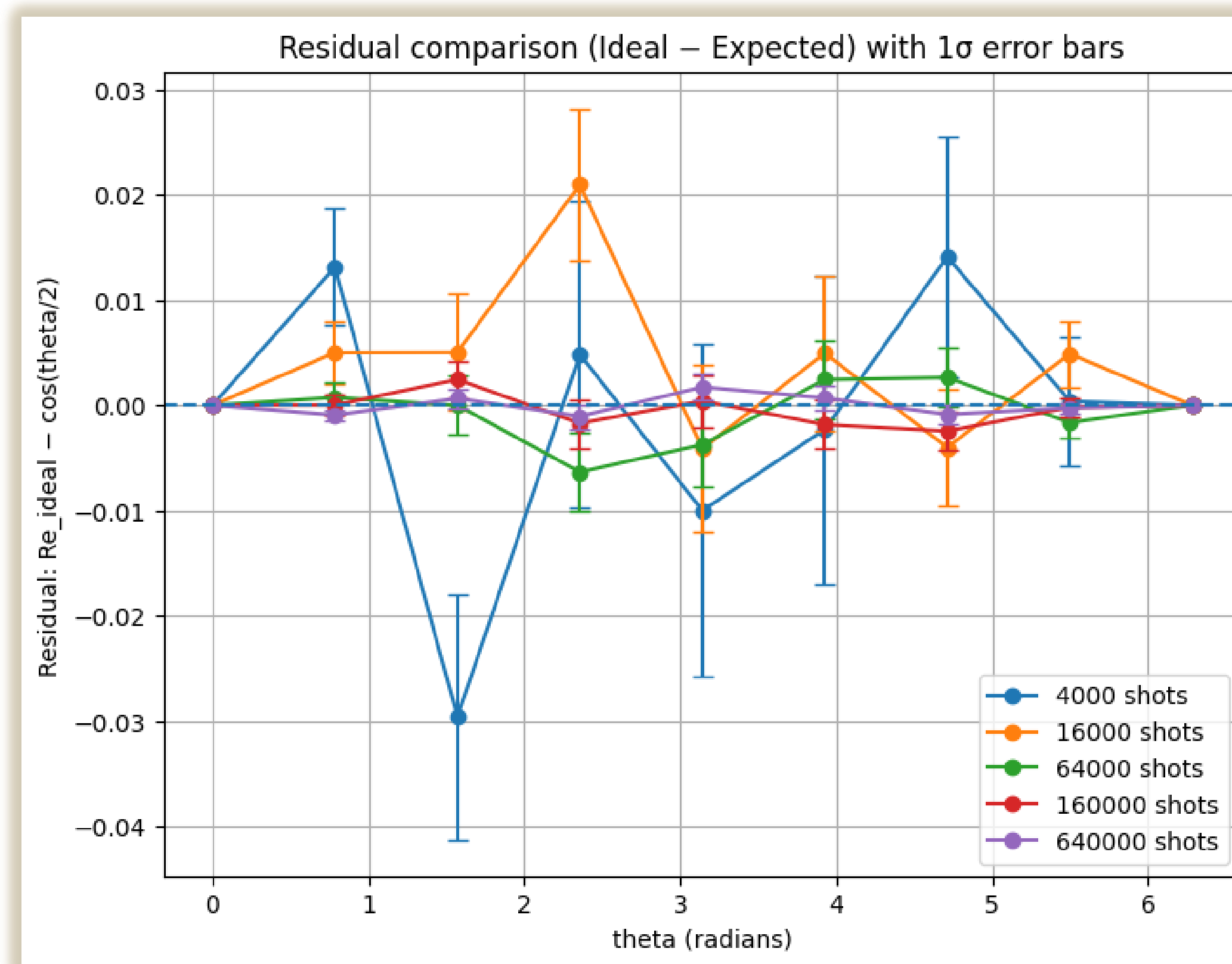
Procedures

- Running circuits in both noisy and ideal simulations at each shot level
- Plotting the residual data at each θ and shot level
- Residuals were plotted with 1σ error bars derived from binomial statistics.

Data Analysis

- Binomial statistics are assumed where $\sigma = 2 \sqrt{\frac{p(1-p)}{N}}$
- Computed Z-scores where $|z| \approx 1$ is within expected statistical fluctuation and $|z| \gg 1$ is larger than expected deviation
- Reduced chi-squared tests
- Variance inflation factor: $\sqrt{\chi^2_{\text{red}}}$ where $k \approx 1$ is purely statistical noise and $k > 1$ is additional noise present

Results



Here our effective uncertainty must be artificially increased to explain observed deviations, confirming excess noise besides binomial shot-noise

Conclusion

Within the IonQ framework ideal simulations obey $\frac{1}{\sqrt{N}}$ scaling. At low shots our noisy framework seems to follow binomial statistics, which contrasts to what is seen on the noisy simulator ("aria-1") which goes beyond binomial trends. By computing reduced chi-square and variance inflation tests we can see that binomial shot-noise is not dominant at high shot counts but rather a systematic and unaccounted for noise that is prevalent.

Significance

This work provides a quantitative framework for directly distinguishing between binomial noise and systematic noise in Hadamard based measurements. Tools such as reduced-chi square and variance inflation factor offer a precise statistical analysis of quantum noise than qualitative methods of analytically approaching interference fringes. Such methods offer a necessary foundation to approaching more complex circuits on quantum computers especially with fully functional non-simulated hardware.

Limitations

Rather than running such tests on a real quantum hardware, a simulation-based approach was chosen so that a semi-rigorous basis could be set for analyzing noise within quantum circuits. This indicates that quantum hardware may not have the same behavior especially in IonQ's quantum framework. Ideal tests on physical hardware impose a higher noisy background to work with and runtimes that scale with the number of shots in both ideal and noisy simulations which may make statistical analysis more difficult to work with.

Future Work

This statistical framework sets the analytical basis for running simulations on physical hardware. Continued studies will include replicating the analysis on quantum hardware and comparing the functionality of both. Such comparisons will also introduce new possible methods of analysis which may be useful in more complex quantum systems. Large multi-qubit entangled systems are of interest as they correspond to dynamic systems involving Quantum Chromodynamics (QCD). Particularly, extending interference tests to systems with multiple Feynman diagrams will be focused on.

References & Acknowledgements

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- Special thanks to Dr. Herschel Chawdhry whose role in quantum computing advancements in high energy physics would have not made this possible