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1-Dimensional Random Walk Simulations

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Introduction

The motion of particles within a lattice can be represented effectively by a biased random walk in more than 1-dimension. Random walks in 1-dimension are known to be modeled by a binomial relationship due to their binary nature. As random walks in more than one dimension can be analyzed as compositions of this, understanding and simulating one-dimensional random walks is crucial to understanding higher dimensional random walks.

Methodology

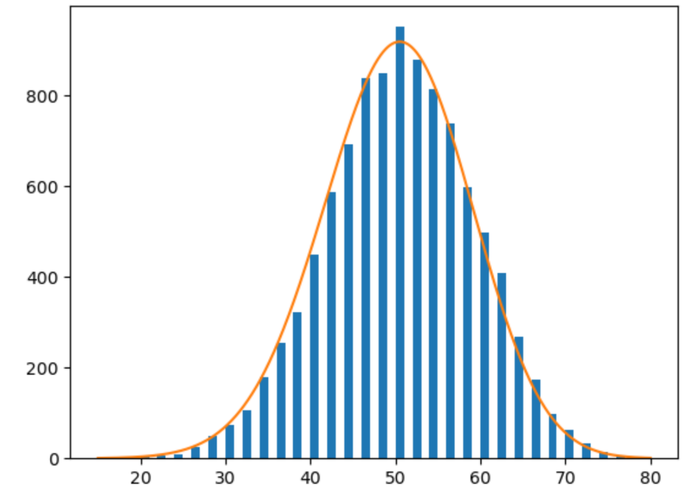
I was tasked by my research mentor with extracting an understanding of programming python and of random walk theory from various python learning and textbooks detailing random walk in order to build an understanding of 1-dimensional random walk. Then, I applied the function

$$k = \frac{1}{2}\left(N + \frac{v}{l}\right)$$

Which relates k, a factor relating the probability of making however many movements to the right, out of N total movements in a random walk of l length (1 in this case). These are plugged in the binomial formula, which applies as there are a limited number of combinations per random walk which would get you a certain value, and thus a random walk can be approximated by a binomial system.

$$p = T\,\mathrm{nCr}\,(N,k)q^k(1-q)^{(N-k)}$$

To verify the random walk formulas, I coded in python formulas which under specific probabilities produce a real random walk. These were graphed onto a histogram, whereas the formula was graphed for final position as a function of the range of the function, showing off all possible values that the random walk should theoretically produce. Results were then compared. Successive simulations of 10,000 Random Walks were done for 100, 1,000, and 10,000 movements



A simulation of 10,000 biased random walks where the chance to move right is 75%. The binomial function is then posted over the histogram for comparison, where the x value is the final position.

Data

The provided function effectively modeled the histograms, showing alignment between current theory on random walks and functional simulations. The current simulation is able to model not only unbiased random walks, but it effectively simulates biased random walks as well.
Convergence to the expectations of random walks was noticed for simulations
The results suggest that the binomial formula is an effective way to represent random walks along a single dimension.