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A Comparative Analysis: Solving Differential Equations with LSTM and Attention-Based Neural Networks

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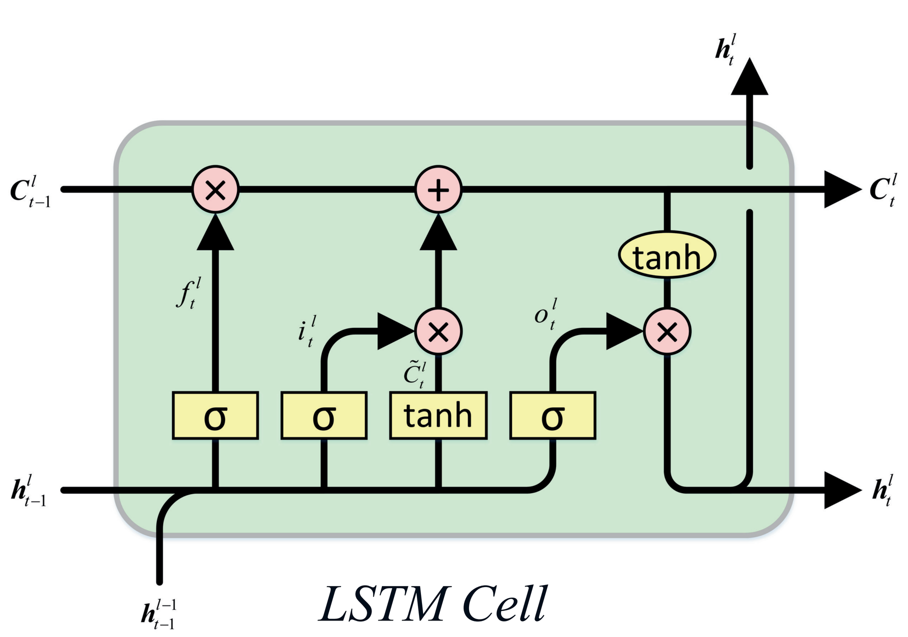
Abstract

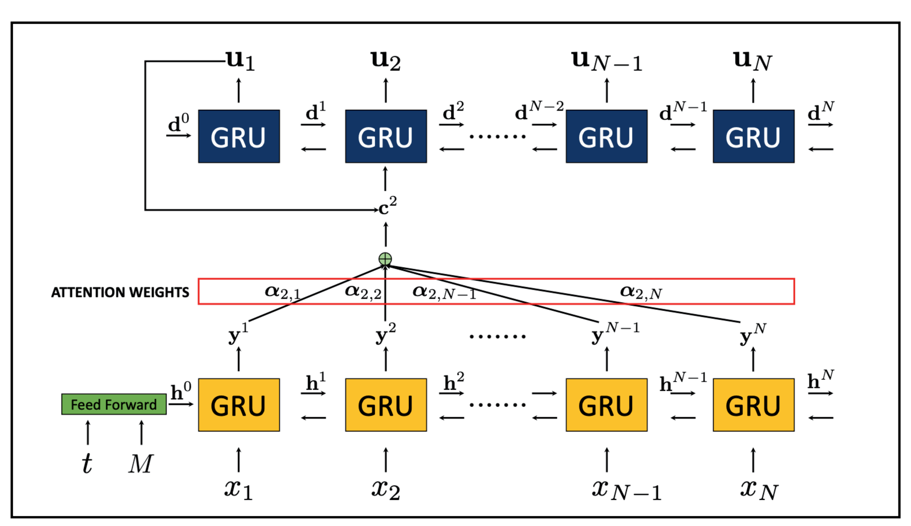
Neural Networks (NNs) are mimic human cognition, through computers, to complete tasks.
•We are interested in Long-Short Term Memory (LSTM) [2] and Attention-Based [8] models
- designed for time series data (over long periods)

•Comparing LSTM and Attention for the purpose of at learning early time differential equation (equations that model change over time) solutions
- Then making long term predictions

•These solutions are relevant to solving for cryogenic fuel tank sloshing dynamics
NNs learn from early solutions to predict long-term behavior.
•Solving differential equations is traditionally done analytically [14] [6] [4] [11].
•In order to do a controlled experiment to see if attention based NNs are better than LSTM, for time series analysis, we utilize substitute a prototype DE (Differential Equation) simulation of cryogenic tanks
•Our research was conducted through literature review, analyzing code and altering parameters to test solution behavior.
•Our results will provide insight on the ability of neural networks to solve problems that are very challenging to solve



LSTM Cell



Attention-Based Neural Network [10]

Introduction

- Direct numerical simulations of cryogenic tank dynamics involves numerically solving a two or three-dimensional Partial Differential Equation (PDE)
  - Takes an unfeasible amount of computer time for realistic simulations.
- Alternative approaches learn from early time data
- Replaces solving a PDE with solving a much simpler Ordinary Differential Equation (ODE)
- In this research, we explore more general methods for learning early time dynamics in order to efficiently predict long term behaviour.
- The advantage of a general approach is that
  - We aren't restricted to tank problems and
  - The general approach enables us to make predictions when the liquid and gas in a tank become "turbulent."

2. Methods

- Trained the NN with the analytical solution to the ODE below
- Set up (and write) code, download necessary extensions (tensorflow, keras, etc.)
- Run code
- Parameters alterred: UNITS, NUM_DENSE
- t=1, dt=0.01, window=50, n_traj=50
- Alter parameters to gain insight

$$\zeta = \frac{1}{20},\ \omega_0 = 2\pi,\ v(0) = v_0,\ x(0) = x_0$$

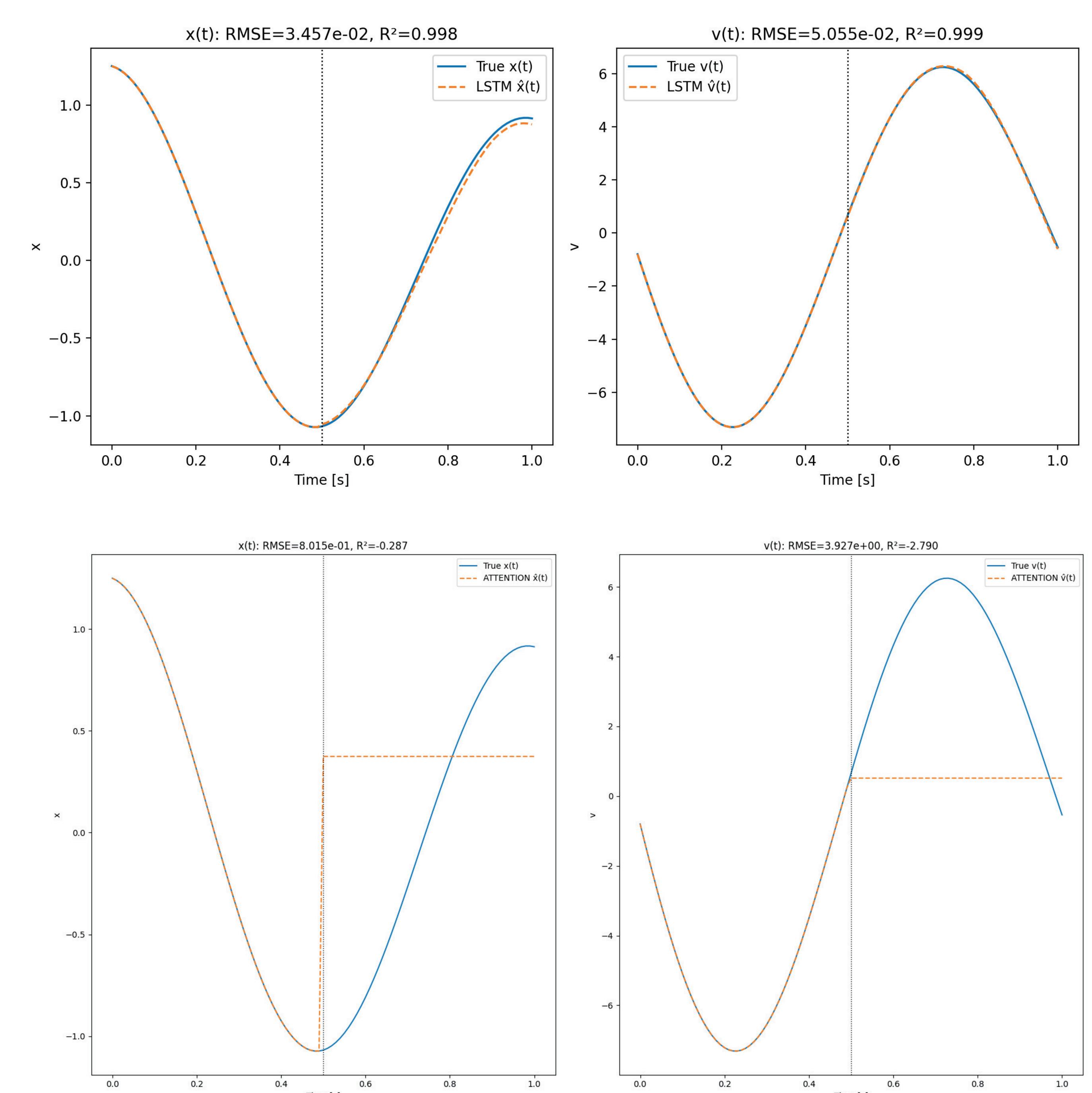
$$\frac{dv}{dt} = -2\zeta\omega_o v(t) - \omega^2 x(t)\ for\ 0 \le t \le T$$

$$\frac{dx}{dt} = v(t)\ for\ 0 \le t \le T$$

Results

LSTM
- UNITS=32, NUM_DENSE=24
  - x(t): $R^2 = 0.945$
  - v(t): $R^2 = 0.863$
- UNITS=64, NUM_DENSE=48
  - x(t): $R^2 = 0.997$
  - v(t): $R^2 = 0.983$
- UNITS=128, NUM_DENSE=96
  - x(t): $R^2 = 0.998$
  - v(t): $R^2 = 0.999$

ATTENTION-BASED
- UNITS=32, NUM_DENSE=24
  - x(t): $R^2 = -0.021$
  - v(t): $R^2 = -12.881$
- UNITS=64, NUM_DENSE=48
  - x(t): $R^2 = -0.676$
  - v(t): $R^2 = -4.654$
- UNITS=128, NUM_DENSE=96
  - x(t): $R^2 = -0.287$
  - v(t): $R^2 = -2.790$



Code Output:
LSTM vs. Actual DE Results Long-Term

Conclusion

To answer which NN is most effective, we concluded the following.
- **LSTM is highly accurate**, increasingly so for higher NUM_DENSE and UNITS values
- **Attention is highly inaccurate** for all parameter values
- Thes significance of these findings is their application to many fields
  - Engineering and Sciences
- Selecting an effective NN will assist in problems of the future.
- Our results are ongoing.

Limitations
- We only assesed LSTM and Attention-Based Neural Networks
- The Attention NN code has a bug

Future Steps
- Extending research to other Neural Network Architectures
- Reworking Attention-Based Code

References

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