Physics-Informed Neural Networks: A Deep Learning Framework for Solving Differential Equations

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Research Objectives

- Develop a Physics-Informed Neural Network (PINN) framework to solve complex equations governing metal 3D printing processes, ensuring accurate modeling of the underlying physics.
- Compare the performance of PINNs with traditional numerical methods, such as Finite Element Methods (FEM), in terms of solution accuracy, computational cost, and scalability for high-dimensional problems.
- Assess the accuracy and computational efficiency of PINNs by analyzing their ability to capture key physical phenomena, optimize training strategies, and generalize across different printing conditions.

Physics Informed Neural Networks

- **Bridging Physics and Machine Learning**: PINNs incorporate known physical laws into neural network training, ensuring solutions adhere to governing equations.
- **Efficient Solution for PDEs**: Traditional numerical methods can be computationally expensive, while PINNs provide an alternative that leverages deep learning for efficiency.
- **Data-Efficient Approach**: PINNs require fewer data points compared to purely data-driven models, as they rely on physics-based constraints to guide learning.
- **Versatility Across Domains**: PINNs are applicable to various scientific and engineering problems, including fluid dynamics, material science, and medical imaging.

Methods

- **Focus:** Using PINNs to solve Differential Equations, comparing them with traditional numerical methods. Specifically, the 1-dimesonal Viscous Burgers Equation
- **Materials:** PINNs trained using supervised learning, backpropagation, and gradient descent.
- **Error metric:** Mean-Squared Error loss combining loss functions from differential equations, boundary conditions, and initial conditions.
- **Training process:** Auto-differentiation computes derivatives to enforce physical laws. Initial and boundary conditions incorporated into loss function. Model iteratively trained to balance data-driven predictions with physical constraints.
- **Benchmark Methods:** Comparison with state-of-the-art numerical techniques, including Finite Element Methods (FEM), to assess accuracy and computational efficiency. The evaluation includes convergence rates, error analysis, and resource usage.

CASE STUDY: 1-Dimensional Viscous Burgers Equation

Boundary Conditions: $u(x \pm 1, t) = 0$
Initial Conditions: $u(x, 0) = -\sin(\pi x)$

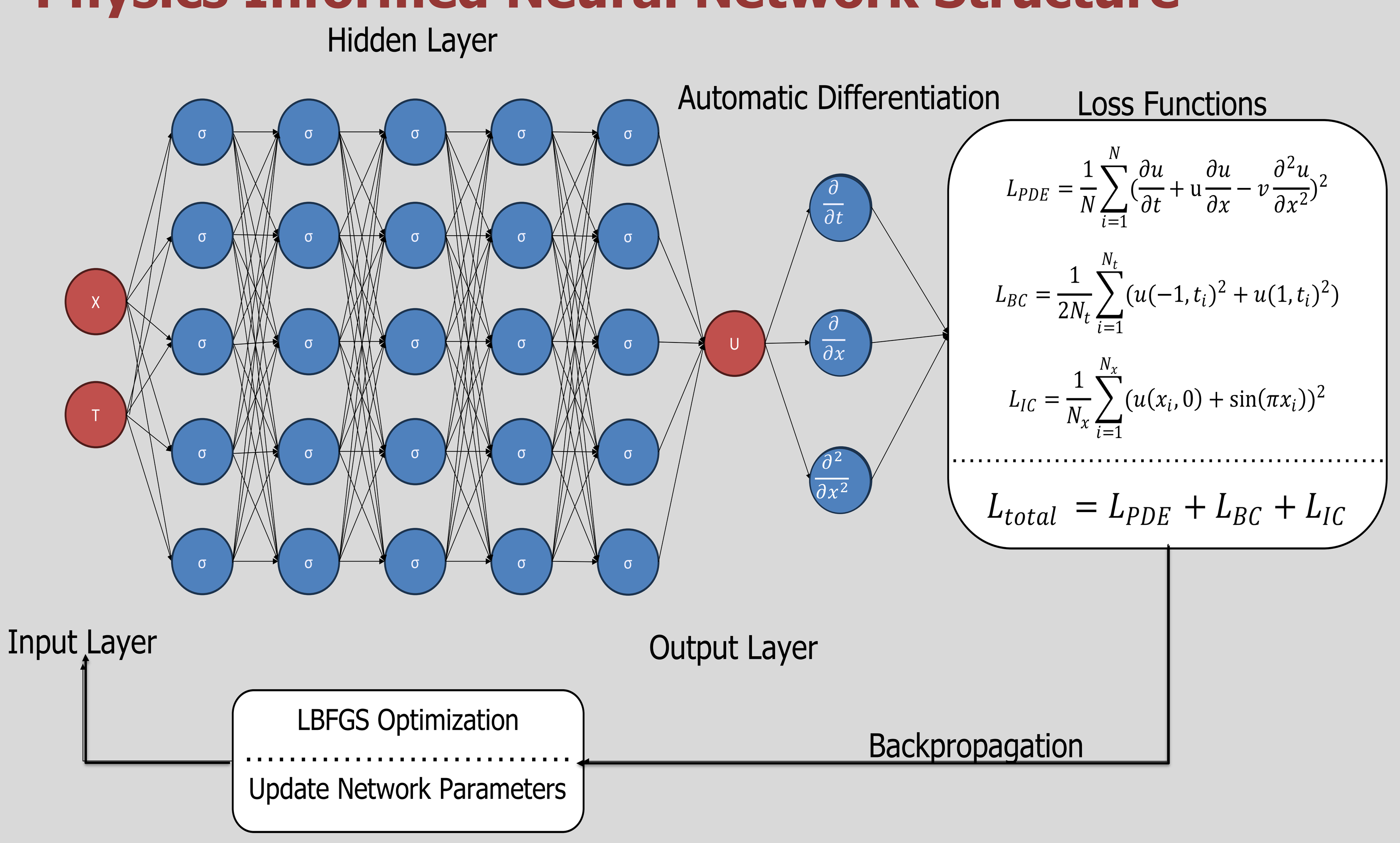
$$\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x} = v\frac{\partial^2 u}{\partial x^2}$$

- u is the velocity
- t is time
- x is the spatial coordinate
- v is the kinematic viscosity ($v > 0$)

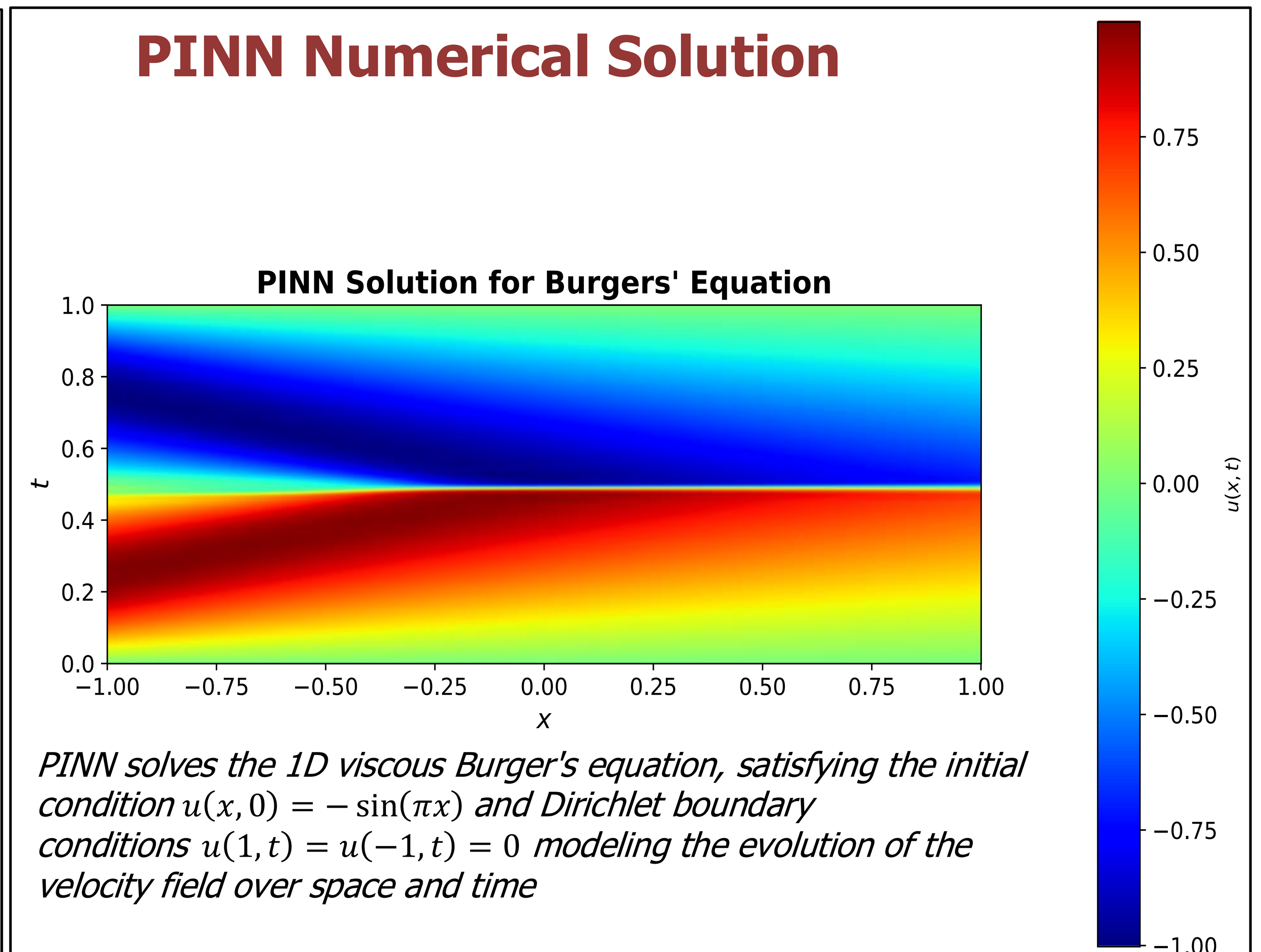
•A fundamental partial differential equation that occurs in various areas of applied mathematics, including fluid mechanics, nonlinear acoustics, gas dynamics, and traffic flow.

•Burgers' equation combines nonlinear wave motion with linear diffusion, making it the simplest model for analyzing the combined effects of nonlinear advection and diffusion.

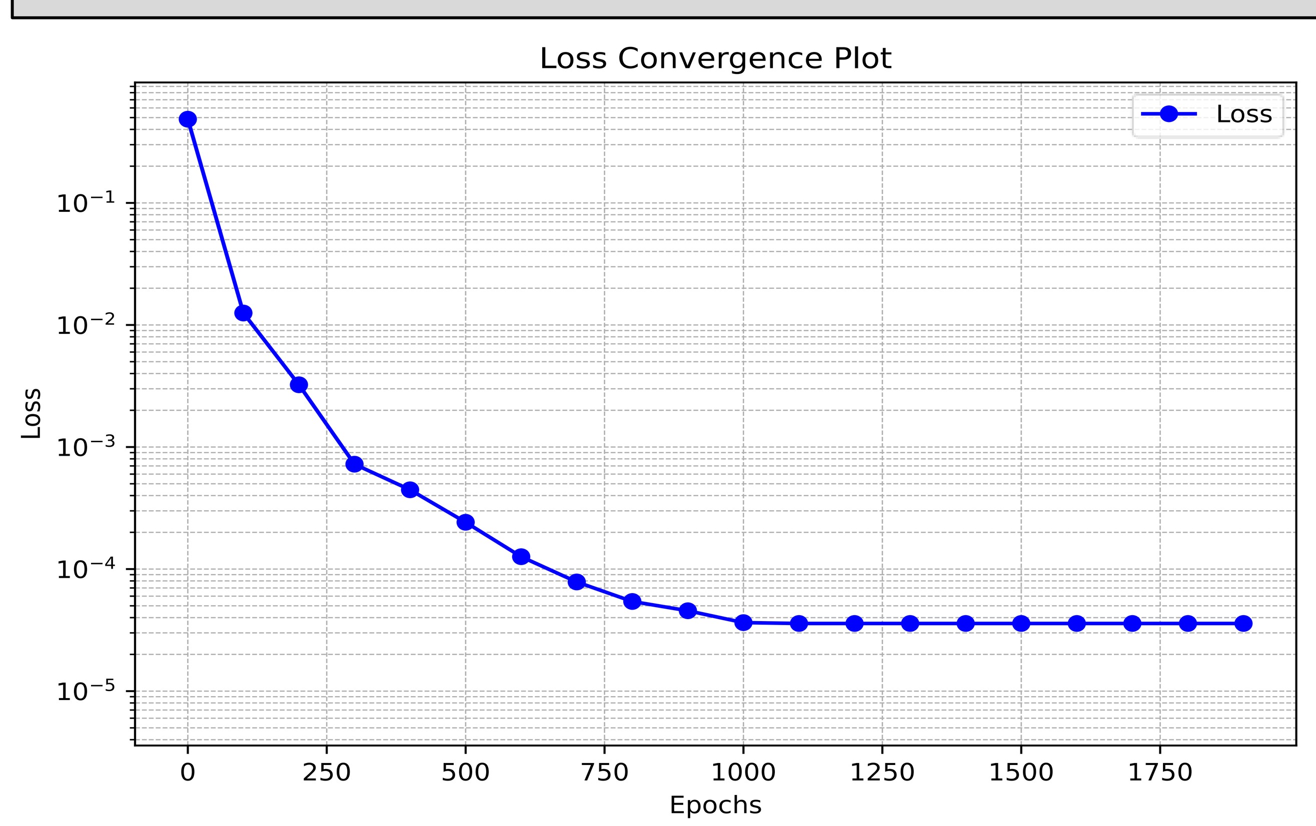
Physics Informed Neural Network Structure



PINN Numerical Solution



PINN solves the 1D viscous Burger's equation, satisfying the initial condition $u(x, 0) = -\sin(\pi x)$ and Dirichlet boundary conditions $u(1, t) = u(-1, t) = 0$ modeling the evolution of the velocity field over space and time



- *Training performance shows significant loss reduction over 2000 epochs, decreasing from 0.4852 to 3.58e-5, demonstrating convergence.*

Conclusions and Future Work

- **Convergence and Performance:** Rapid initial loss reduction with convergence around epoch 1100, maintaining stability thereafter. The **Limited-memory Broyden–Fletcher–Goldfarb–Shanno (LBFGS)** optimizer effectively minimized loss, ensuring a good fit to the physics and specified conditions.
- **Conclusions:** Summarized findings highlight the effectiveness of PINNs in solving PDEs, demonstrating accuracy and computational efficiency.
- **Next Steps:** Future work involves extending the PINN framework to solve multi-physics equations, specifically the heat equation for laser powder bed fusion.

References:

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Code Repository: