Undefined: How do People Understand the Concept of Zero?

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Background

- Many students and teachers do not know that *n*/0 is *undefined,* or why
- Intuitive "concrete" arguments may mislead
  - e.g., If you distribute 5 donuts evenly across 0 plates, how many donuts are on each plate?
- "Formal" arguments more logically sound
  - e.g., If 5/0 = *x*, then *x**0 = 5; no number satisfies *x*
- High school students who know n/0 is undefined less likely to accept validity of formal arguments vs. concrete
- More research on this topic may help improve teaching about division by 0

Our Study

Research Question 1: How accurate and confident are undergraduates on division by 0 and by other numbers?

Research Question 2: Does cognitive reflection tendency predict accuracy on division by 0 problems?

Research Question 3: Which variables (e.g., argument type, validity, or subject) are related to argument convincingness?

References

Method

Participants

- 46 FSU Undergrads (16 men, 30 women)
  - 76% STEM majors
  - 39% currently enrolled in a math course

Materials & Procedure

Participants completed the following tasks on Qualtrics (~30 min):

- **Arithmetic test**
  - 2 problems of each type: n/0, 0/n, 0/0, n/m, n*0, n*m
  - 5-point likert scale self-rated confidence for each problem
- **Cognitive Reflection Test** (Frederick, 2005)
  - 7 questions designed to quickly evoke an intuitive but incorrect answer
    - E.g., If you're running a race and you pass the person in 2nd place, what place are you in now?
- **Arguments**
  - 8 statements of solutions to division problems (2 [true/false] x 4 [problem types: *n*/0, 0/*n*, 0/0, *n*/m])
    - Agree/disagree
  - 4 arguments per statement (2 formal [Div. as inverse of mult. and Div. as repeated subtraction] + 1 concrete)
    - 3-point scale convincingness rating

True Statement Example:

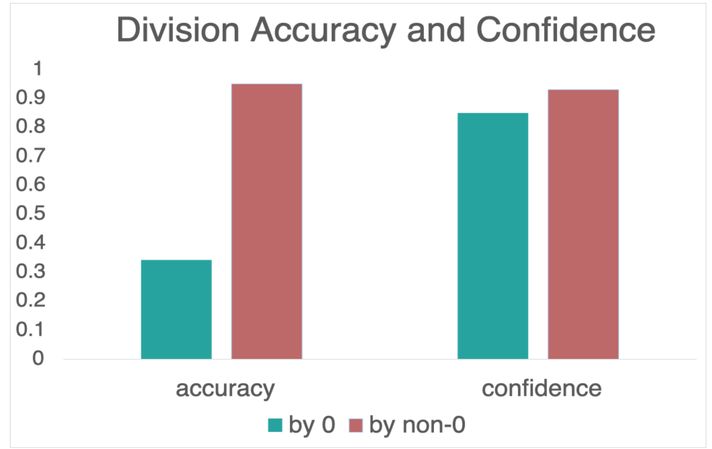
- Alex, Myah, Sherane, and Tiffany stated that **0 / 4 = 0.**
- Alex writes: "**Division is the opposite of multiplication.** So if 0 / 4 = x, then 4x = 0, and x must be 0. Therefore, 0 / 4 = 0."
- Myah writes: "**Division is like repeated subtraction.** The answer to 0 / 4 is the number of times we can subtract 4 from 0 before reaching 0. Since 0 is the starting point, we can subtract 4 zero times before reaching 0. Thus, 0 / 4 = 0."
- Sherane writes: "**Division is like sharing.** If there are 4 people with 0 cookies to share, then each person gets 0 cookies. Therefore, 0 / 4 = 0."
- Tiffany writes **"It's a rule** of math that 0 divided by any nonzero number equals 0. Therefore, 0 / 4 = 0."

False Statement Example:

- Hugo, Marceline, Katherine, and Mohammed say that **3 / 0 = 0.** Do you agree with their statement?
- Hugo writes "**Multiplication is the opposite of division,** so if 3 / 0 = x then 3x = 0. Only 0 can replace 'x' to make 3x = 0 true, so we can conclude that 3 / 0 = 0."
- Marceline writes "**Division can be modeled as repeated subtraction.** The solution to 3 / 0 is the number of times 3 needs to be subtracted from 0 to reach 0. Since 3 doesn't need to be subtracted from 0 to reach 0, we can conclude that 3 / 0 = 0."
- Katherine writes "**Division can be modeled as sharing.** Solving 3 / 0 is like figuring out how many flowers go in each vase if we have 3 vases and 0 flowers. There will be 0 flowers in each vase, so we can conclude that 3 / 0 = 0."
- Mohammed writes "**It's a rule** of mathematics that any number divided by 0 equals 0. Therefore, 3 / 0 = 0."

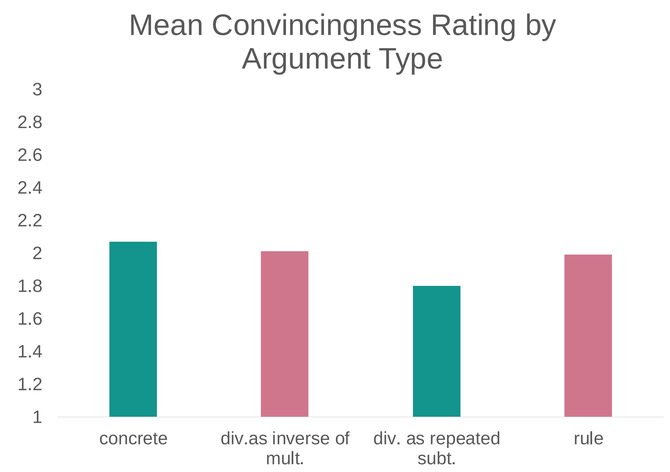
Preliminary Results

RQ 1: Participants were significantly more accurate and confident with division by nonzero *n*s than by zero



RQ 2: There was ***not*** a significant correlation between CRT and division by 0 accuracy.

RQ 3: Participants found concrete arguments most convincing and "division as repeated subtraction" formal arguments least convincing



Discussion

- Further analyses are necessary to understand the relations between argument type, cognitive reflection, arithmetic knowledge, and argument evaluations