Fractal Dimension of Thermonuclear Deflagrations in Type Ia Supernovae

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Introduction

Type Ia supernovae (SN Ia) are the largest thermonuclear explosions in the universe. They are created when a white dwarf star (WD) accretes matter, usually from a companion star. As this material accumulates on the WD surface, it may become dynamically unstable and collapse if its mass reaches a critical mass of ~1.4 solar masses, known as the Chandrasekhar mass limit. While approaching that critical mass, theoretical models indicate stellar core compresses, thermonuclear reactions start, and convection develops transporting heat from the core to about half of the WD radius. Computational models also demonstrated that as the thermonuclear heating intensifies a small amount of mass may undergo ignition resulting in formation of a subsonic burning front known as a deflagration. Those flame fronts initially increase their size by advancing radially, but quickly become unstable as the buoyancy force due to gravity leads to additional acceleration of flame front segments more extended in radius due turbulence naturally existing in stellar interiors. This process is known as the Rayleigh-Taylor instability (RTI), in which a lighter, expanded due to burning thermonuclear ashes tries to support denser, unburned fuel against gravity. RTI results in strong deformation of the flame surface. In numerical simulations, complexity of the flame front appears increasing with the increasing numerical resolution. Because an increase in the flame surface area results in an increases of the effective flame speed, this puts in question numerical convergence of RTI computer models. This problem could be addressed if there exists a universal scaling relation between flame surface area and physical scale.

In the case of RTI those RTI unstable systems seem contain self similar patterns in terms of scale like those displayed in figure 1. In computational geometry, objects which display this kind of behavior are called fractals. This motivates the analysis of the RTI unstable systems in terms of their fractal characteristics.

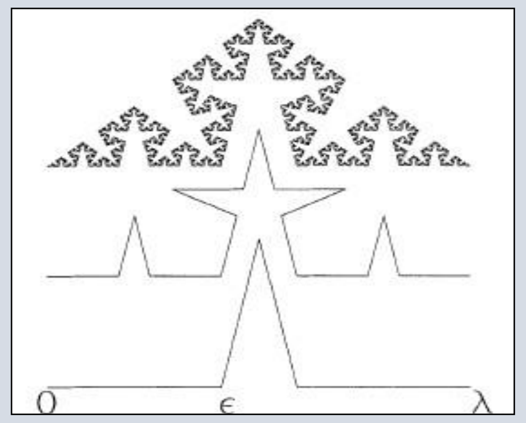


Figure 1: Illustration of a system with self-similar geometrical structure. The same object seen at various resolutions is shown in three panels. Fractal dimension assumes there exists a relation between the geometrical complexity and scale. Adapted from [1].

Estimation of fractal dimension in RTI models, permits to quantify and characterize their geometrical complexity. This motivates analysis of various physical systems in which RTI pays an important role but have different participating physics. Here we consider examples of pure hydrodynamics, RTI unstable fame front and the thermonuclear deflagration in the SN Ia explosion. In this project our focus is on the explosion mechanism of the Type Ia supernovae.

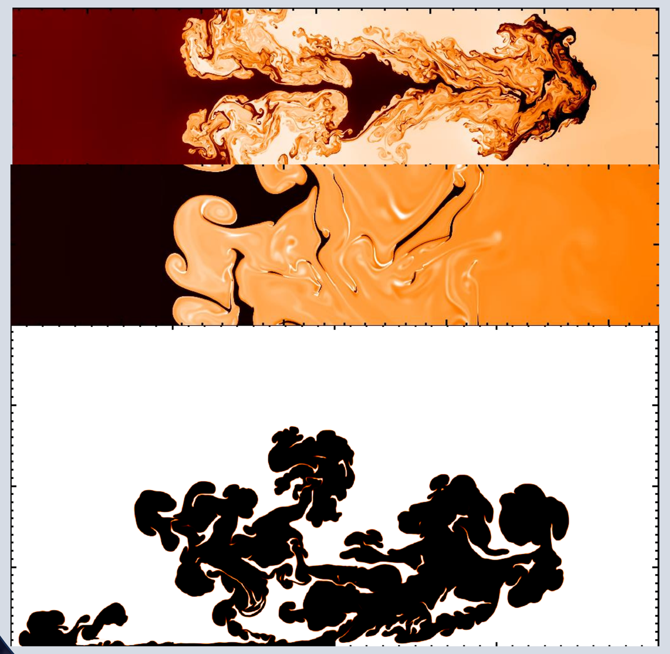


Figure 2: Structure of RT-unstable interfaces in pure hydro model (top panel), SN Ia flame model (middle panel), and SN Ia explosion model (bottom panel). Those models differ in terms of participating physics, such as thermonuclear combustion and self-gravity. See text for details.

Methods

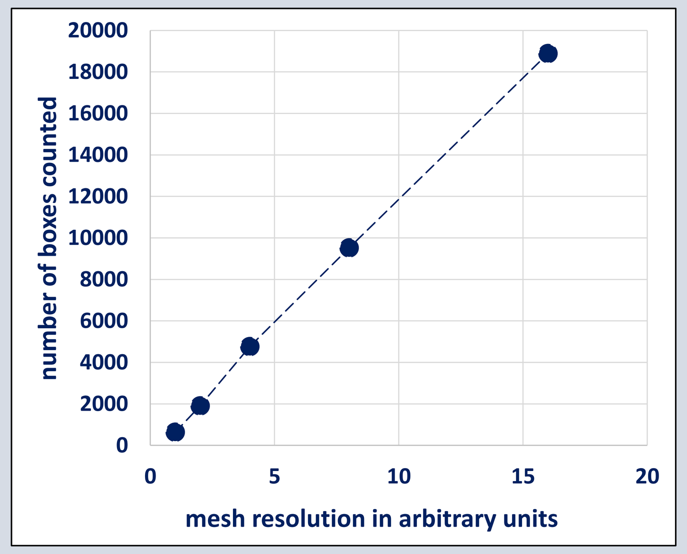
The simulation results used this study were obtained using Proteus code which is a computational tool designed for studying complex Multiphysics problems including Rayleigh Taylor Instability (RTI). Proteus solves a set of equations relevant to applications such as hydrodynamics, self gravity, and nuclear burning. The output of the code includes information about solution variables such as density of the fluid, velocity, and temperature. This information is stored for future analysis in a series of output files which can be analyzed using appropriate software packages. In this study we used two approaches to estimate fractal dimension, one being a MATLAB program and the second being a C++ collection of modules for analysis of Proteus type output called LAVAflow [5]. One of these LAVAflow modules allows fractal dimension to be computed [2].

In this project we use Proteus code to study applications such as pure hydrodynamic model of RTI, a model of RTI unstable thermonuclear deflagration, and an integrated type 1a supernovae.

- Pure hydrodynamic – requires hydrodynamics and constant gravity
- Thermonuclear deflagration – requires hydrodynamics, constant gravity, and a model that describes the evolution of the flame which includes changes in isotopic composition of the matter and release of thermonuclear energy.
- SN Ia Explosion – requires hydrodynamics, self gavity, and a model of the flame like in thermonuclear deflagration.

The LAVAflow package was created in order to develop a set of analysis tool in the form of computational modules to calculate properties of the solution produced by the Proteus code. In the context of this project the required module calculates the fractal dimension using two inputs: a solution variable and its value. This module uses the box counting method to estimate fractal dimension. In the box counting method the complexity of the structure is characterized by the number of elements required to represent that structure at a given resolution. By calculating the number of elements at different resolution you drive an algebraic relation between the two which can be expressed as $N = \left(\frac{1}{\Delta x}\right)^D$ where D is the fractal dimension, Δx is the size of the boxes, and N is the number of boxes required to cover the fractal object.

Figure 3: Box count dependance on scale in SN Ia flame model. The variable used is the flame progress variable and the value defining the flame position is set to 0.5. This data are used to estimate the fractal dimension using the box counting method. See text for details.



Results

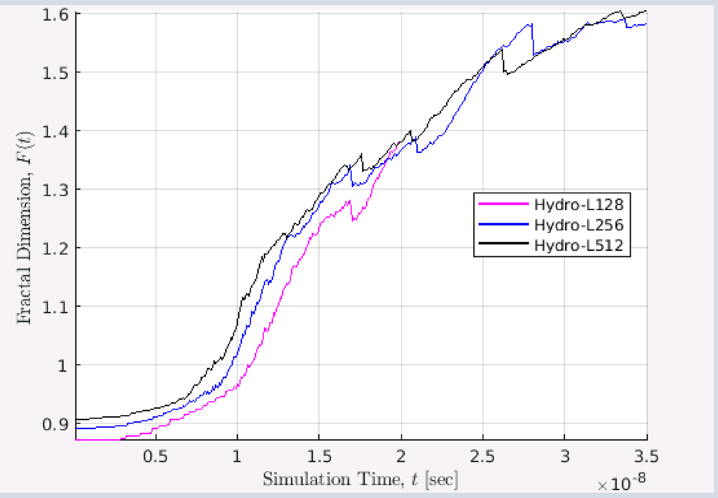


Figure 4: Evolution of the fractal dimension in three hydro models. The models differ in terms of their initially imposed velocity perturbation's.

Figure 4 shows the evolution of fractal dimension in the pure hydro models initially has a value close to 1 as the initial conditions for the model has a smooth shape. As the RTI develops and the flame surface becomes progressively more corrugated, the fractal dimension increases to about 1.6.

Figure 5 shows the evolution of fractal dimension in the RT flame models initially has a value close to 1 as the initial flame profile has a smooth geometric structure. As the RTI develops and the flame surface becomes progressively more corrugated, the fractal dimension increases. As the evolution enters a nonlinear phase, the degree of the flame surface stabilizes.

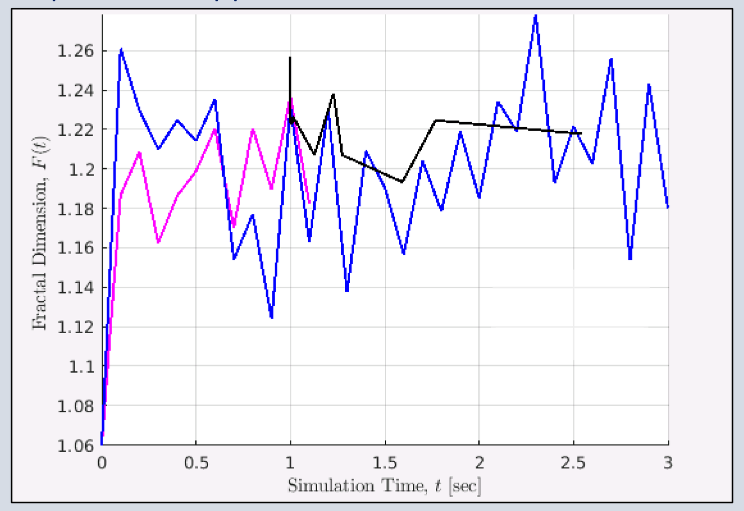


Figure 5: Temporal evolution of the fractal dimension in the SN Ia flame model. The value of fractal dimension is shown for model conditions found in a SN Ia explosion model when the flame reaches fuel of the density 1×10^8 g/cc (pink solid line), 2×10^7 g/cc (black solid line), and 6×10^6 g/cc (blue solid line). See text for details.

Figure 6 shows the evolution of fractal dimension in 3 SN Ia models starting from different initial conditions. Up till around 0.6 seconds where when the curve reaches the curves remain the same because there is not yet any flame but a representation of the artificial creation of the flame. After that, the models separate a bit but continue to closely follow the same trend and increase to a fractal dimension of about 1.3.

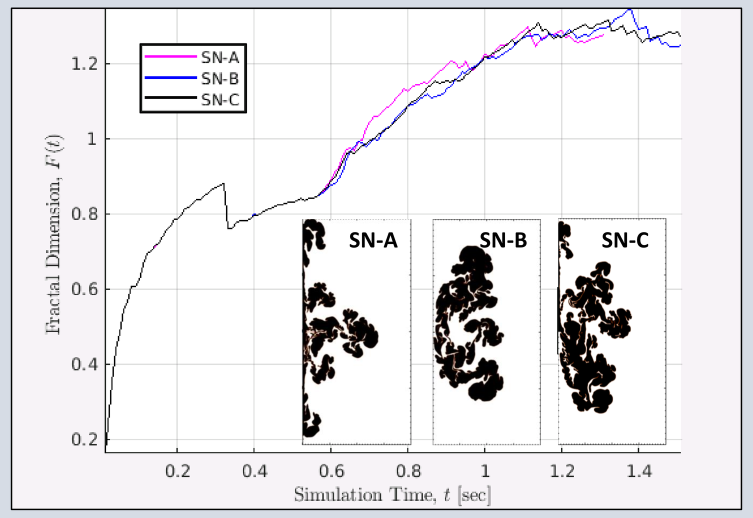


Figure 6: Evolution of the fractal dimension in three SN Ia explosion models. The models only differ in terms of the initially imposed small velocity perturbations. The insets show the value of the flame progress variable, with fully burned material shown in black. The edge of that region corresponds to the location of the flame front, and its structure is analyzed in terms of fractal dimension. See text for details.

Discussion

The rate of which the flame advances determines the characteristics of the supernovae. We used fractal dimension to measure these properties.

In all models the fractal dimension grew from a small value created by smooth initial conditions to a much higher value as evolution progresses due to increasing complexity. We see that even though the pure pure hydro model has the same RT physics as the flame and SN Ia explosion the resulting structure is very different.

In the future we should study 3 dimensional objects that represent the supernovae more accurately. As 2-dimensional turbulence does not have the same properties as 3-dimensional turbulence.

[1] Blinnikov, S.I. & Sasorov, P.V. 1953, 4827.
[2] Brenner, Samuel. 2013. Fractal and Multifractal Analysis as Tools to Characterize Supernovae and Molecular Clouds.
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