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## SIR Model

- William Kermack and Anderson McKendrick created the original SIR model in 1927 where they published their paper titled *A Contribution to the Mathematical Theory of Epidemics*
- It helps predict the spread of disease by separating the population into three groups: Susceptible, Infected, and Recovered.

$$\begin{aligned}\frac{dS}{d\tau} &= -R_0 SI \\ \frac{dI}{d\tau} &= R_0 SI - I \\ \frac{dR}{d\tau} &= I,\end{aligned}$$

Here is the original Kermack & McKendrick Model separated into 3 different related functions to model the Susceptible, Infected, and Recovered populations.

- The overall trend that we want to achieve is a flattened infection curve meaning there is no dramatic spike of infected individuals and the rate of infection is more consistent and spread out over time.
- We achieve this result in certain extensions of the model by individuals' behavior ( $p$ ) changing and favoring implementing higher levels/more NPIs across the population.

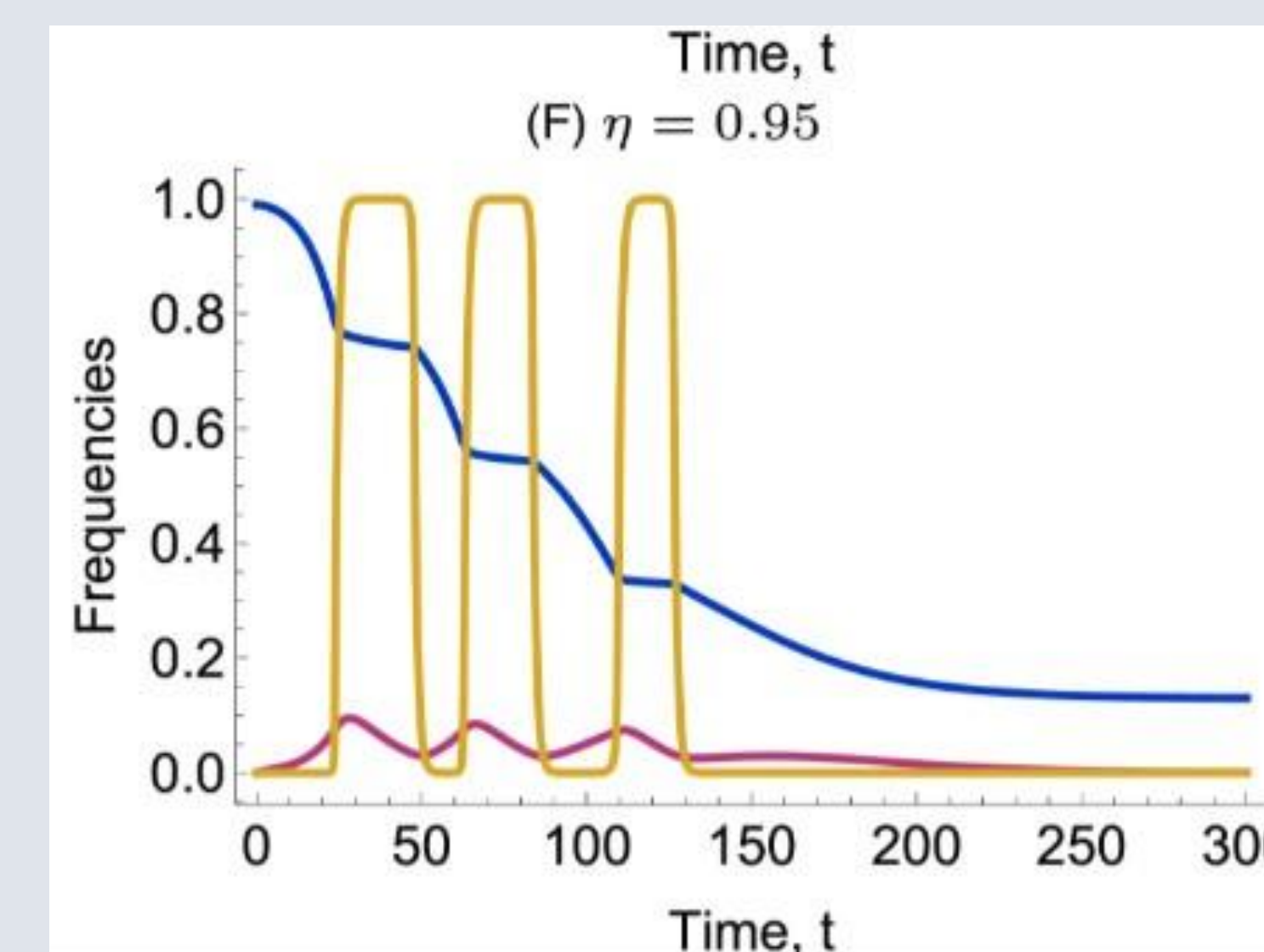
## Extensions of the Model

- There are different modifications researchers have made to the original SIR model to include different variables, such as behavior and how it affects the use of NPIs.
- NPI: Non-Pharmaceutical Intervention such as washing hands, wearing a mask, social distancing, etc.
- Included in the model from *The impact of threshold decision mechanisms of collective behavior on disease spread*, the function of social utility,  $w(p, q) = -\theta(p - q)^2$  used to reflect what affects individuals' decisions and determine if the cost,  $c$ , is worth deviating from the normal behavior of the population.
- Using an NPI comes with a resulting perceived or real cost:  $c$ . This can be a monetary, social factors, and/or physical cost.

Epidemics are worldwide catastrophes that claim many lives each year. To be able to model multiple variables affecting the spread of these diseases could help us understand how to better deescalate the impacts by effectively implementing safety measures and precautions. This literature review covers some variables that affect the rate of infection in a population using compartmental modeling to analyze the dynamics of the epidemic. These show which implementations are most effective and at what time in order to be able to predict which measures can best control the spread of the disease. Using the SIR model (Susceptible, Infected, and Recovered) and SEIR models (Susceptible, Exposed, Infected, Recovered), researchers created differential equations using many different variables representing the possible protective measures people may take to protect against the disease. These equations are then solved numerically and used to project the oscillations and trends of the epidemic. Using these projections derived from the mathematical models, we can help shape public health policies to ensure that the most effective strategies are implemented in society.

**Table 1. Summary definitions of parameters with default values**

Parameter	Definition	Default value
$\beta_0$	Intrinsic transmission rate	0.4/d
$1/\gamma$	Average recovery period	10 d
$1/\delta$	Average latency period	5 d
$\epsilon$	Behavioral change rate	1/d
$\eta$	Efficacy of the NPI	0.8
$\kappa$	Payoff differential sensitivity	1000
$c$	Material cost	0.02
$\theta$	Relational cost	0.01



Above to the left is a table depicting the essential parameters used in the models. To the right is a time series showing the trajectory of the epidemic with the effectiveness of NPI use at 95% (Morsky, et al., 2023).

- In order to determine what course of action should be taken in order to flatten the infected curve, we need to know which variables we need to account for and how to use them in the dynamic models.
- Flattening the curve refers to the curve of the line when graphing the basic baseline function of the infected individuals in the population which is presented as a "hump". When this hump has a greater height, there are more infected individuals.
- When its height is lowered this means that the number of infections is distributed more evenly over a period of time and is more easily managed rather than a sudden spike in infections among the population.
- An increase in NPI effectiveness always results in a lower peak of infected individuals, but can result in more people becoming infected overall because of the more even distribution.

## SEIR(S) Model

- The SEIR(S) model is an extension of the original SIR model.
- $\beta_0$  = intrinsic transmission rate, meaning the average number of other people an infected individual will infect before they are recovered.
- It includes another category of exposed individuals which takes into account the latency period ( $1/\delta$ ) or how long it takes an individual to become infected after they have been exposed to the disease.
- This causes a delay in the infected spikes in the projections in comparison to the original SIR model.
- This model also takes into account asymptomatic individuals who aren't aware that they are infected and can still transmit the disease to others.
- With this model it is possible for individuals to not receive immunity after infection and become susceptible again, hence the second "S".

## References

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